## Section 1.3: Vector functions

Parametric equations:

$$x = x(t), \quad y = y(t)$$

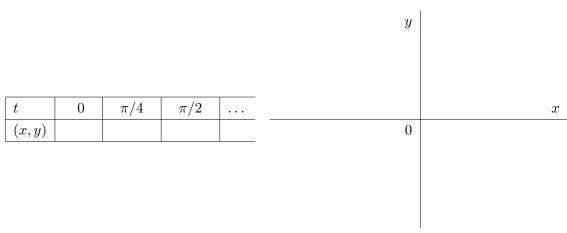
where the variable t is called a **parameter**. Each value of the parameter t defines a point that we can plot. As t varies over its domain we get a collection of points (x, y) = (x(t), y(t)) on the plane which is called the **parametric curve**.

Each parametric curve can be represented as the **vector function**:

$$\overrightarrow{r(t)} = \langle x(t), y(t) \rangle$$
.

Note that Parametric curves have a direction of motion given by increasing of parameter t. So, when sketching parametric curves we also include arrows that show the direction of motion.

EXAMPLE 1. Examine the parametric curve  $x = \cos t$ ,  $y = \sin t$ ,  $0 \le t \le 3\pi/2$ .



EXAMPLE 2. Given  $\mathbf{r}(t) = \langle t+1, t^2 \rangle$ .

- (a) Does the point (4,3) belong to the graph of  $\mathbf{r}(t)$ ?
- **(b)** Sketch the graph of  $\mathbf{r}(t)$ .

$t$ $\mathbf{r}(t)$	y	
-2		
-1		
0		
1	0	
2	U	

(c) Find the Cartesian equation of  $\mathbf{r}(t)$  eliminating the parameter.

EXAMPLE 3. Find the Cartesian equation for  $\mathbf{r}(t) = \cos t \mathbf{i} + \cos(2t) \mathbf{j}$ 

EXAMPLE 4. An object is moving in the xy-plane and its position after t seconds is given by  $\mathbf{r}(t) = \langle 1 + t^2, 1 + 3t \rangle$ .

- (a) Find the position of the object at time t = 0.
- **(b)** At what time does the object reach the point (10, 10).

(c) Does the object pass through the point (20, 20)?

(d) Find an equation in x and y whose graph is the path of the object.

A Vector equation of the line passing through the point  $(x_0, y_0)$  and parallel to the vector  $\mathbf{v} = \langle a, b \rangle$  is given by

$$\mathbf{r} = \mathbf{r_0} + t\mathbf{v},$$

where  $\mathbf{r_0} = \langle x_0, y_0 \rangle$ .

The vector equation of the line can be rewritten in parametric form. Namely, we have

$$\begin{aligned} \langle x(t), y(t) \rangle &= \mathbf{r} = \mathbf{r_0} + t\mathbf{v} = \\ &= \langle x_0, y_0 \rangle + t \langle a, b \rangle = \langle x_0, y_0 \rangle + \langle ta, tb \rangle = \\ &= \langle x_0 + ta, y_0 + tb \rangle \,. \end{aligned}$$

This immediately yields that the **parametric equations of the line** passing through the point  $(x_0, y_0)$  and parallel to the vector  $\mathbf{v} = \langle a, b \rangle$  are

$$x(t) = x_0 + at$$
,  $y(t) = y_0 + bt$ .

EXAMPLE 5. Find parametric equations of the line

(a) passing through the point (1,0) and parallel to the vector  $\mathbf{i} - 4\mathbf{j}$ ;

**(b)** passing through the point (-4,5) with slope  $\sqrt{3}$ ;

(c) passing through the points (7,2) and (3,2).

EXAMPLE 6. Determine whether the lines  $\mathbf{r}(t) = \langle 1+t, 1-3t \rangle$ ,  $\mathbf{R}(s) = \langle 1+3s, 12+s \rangle$  are parallel, orthogonal or neither. If they are not parallel, find the intersection point.