

Section 3.4: Derivatives of Trigonometric Functions

It is important to remember that everything for six trigonometric functions ($\sin x$, $\cos x$, $\tan x$, $\cot x$, $\csc x$, $\sec x$) will be done in radians.

EXAMPLE 1. *Compute:*

$$\text{(a)} \lim_{x \rightarrow 0} \sin x =$$

$$\text{(b)} \lim_{x \rightarrow 0} \cos x =$$

THEOREM 2.

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1,$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0.$$

Proof

EXAMPLE 3. *Find these limits:*

$$\text{(a)} \lim_{x \rightarrow 0} \frac{\sin(5x)}{x}$$

$$\text{(b)} \lim_{x \rightarrow 0} \frac{\sin(9x)}{\sin(7x)}$$

$$\text{(c)} \lim_{x \rightarrow 0} \frac{x}{\sin(4x)}$$

Conclusion: If $a, b \neq 0$ then

$$\lim_{x \rightarrow 0} \frac{\sin(ax)}{x} = \quad ,$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin(ax)} = \quad ,$$

$$\lim_{x \rightarrow 0} \frac{\sin(ax)}{\sin(bx)} =$$

$$(d) \lim_{x \rightarrow 0} \frac{1}{x^2 \cot^2(3x)}$$

$$(e) \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x}$$

EXAMPLE 4. Find the following derivatives:

$$(a) \frac{d}{dx} \sin x =$$

Remark Similarly one can get $(\cos x)' = -\sin x$.

$$(b) \frac{d}{dx} \tan x =$$

Derivatives of Trig Functions (memorize these!)

$\frac{d}{dx} \sin x =$	$\frac{d}{dx} \cos x = -\sin x$	$\frac{d}{dx} \tan x =$
$\frac{d}{dx} \csc x = -\csc x \cot x$	$\frac{d}{dx} \sec x = \sec x \tan x$	$\frac{d}{dx} \cot x = -\csc^2 x$

EXAMPLE 5. Find the derivative of these functions.

(a) $y = \cot x + 5 \sec x + x\sqrt{x}$

(b) $f(x) = \frac{\cos x}{1 + \sin x}$

EXAMPLE 6. Find the equation of the tangent line to the graph of function $y = x^2 \sin x$ at $x = \frac{\pi}{4}$.

EXAMPLE 7. *Find where the tangent line to the graph of $y = x + 2 \sin x$ is horizontal.*