

3.9: Slopes and tangents of parametric curves

Consider a curve C given by the parametric equations

$$x = x(t), \quad y = y(t),$$

or in vector form:

$$\mathbf{r}(t) = \langle x(t), y(t) \rangle.$$

If both $x(t)$ and $y(t)$ are differentiable, then

$$\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle$$

is a vector that is tangent to C . Its slope is:

$$\text{slope} =$$

Another way to see this is by using the Chain Rule. We have $y = y(x(t))$ and then

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}$$

which implies

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

EXAMPLE 1. Find the equation of tangent line to the curve

$$x(t) = \sin t, \quad y(t) = \tan t$$

at the point corresponding to $t = \frac{\pi}{4}$.

EXAMPLE 2. Find the equation of tangent line to the curve

$$x(t) = t + 1, \quad y(t) = t^2 + 4$$

at the $(2, 5)$.

EXAMPLE 3. Find the points on the curve

$$x = t + t^2, \quad y = t^2 - t$$

where the tangent lines are horizontal and there they are vertical.

EXAMPLE 4. *Show that the curve*

$$x = \cos t, \quad y = \cos t \sin t$$

has two tangents at $(0, 0)$ and find their equations.