

## 4.4: Derivatives of Logarithmic Functions

EXAMPLE 1. *Using Implicit Differentiation find the derivatives of the following function:*

(a)  $f(x) = \ln x$

(b)  $f(x) = a^x$

Combining the formulas obtained in Example 1 and Chain Rule one can get

$$\frac{d}{dx} \ln(g(x)) =$$

and

$$\frac{d}{dx} a^{g(x)} =$$

EXAMPLE 2. *Find the derivative:*

(a)  $f(x) = \ln(\sin x)$

(b)  $f(x) = \ln(1 + \ln(1 + \ln x))$

(c)  $f(x) = \ln|x|$

(d)  $f(x) = \cot^3 x + 3^{\cot x}$

EXAMPLE 3. Using the change of base formula, find the derivative formula for  $f(x) = \log_a x$  and generalize it using the Chain Rule.

EXAMPLE 4. Find the derivative of  $f(x) = \log_2(3 + x^2 + x^3)$

**Logarithmic Differentiation** can be used to find derivative of complicated functions involving products, quotients or powers.

*STEPS IN LOGARITHMIC DIFFERENTIATION:*

1. Take logarithms of both sides of an equation  $y = f(x)$  and simplify (f.ex. split a product or quotient, etc.).
2. Differentiate implicitly with respect to  $x$ .
3. Solve the resulting equation for  $y'$ .
4. Plug in  $y = f(x)$ .

EXAMPLE 5. Find the derivative:  $y = (\cos x)^{\sin x}$