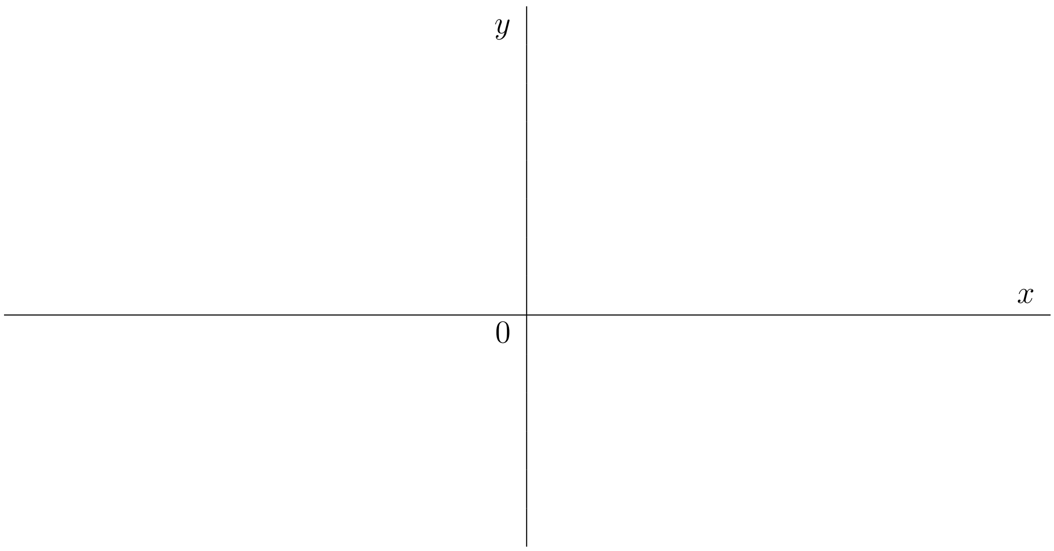
5.1: What does $f'(x)$ say about f?

What does $f'(x)$ say about f?

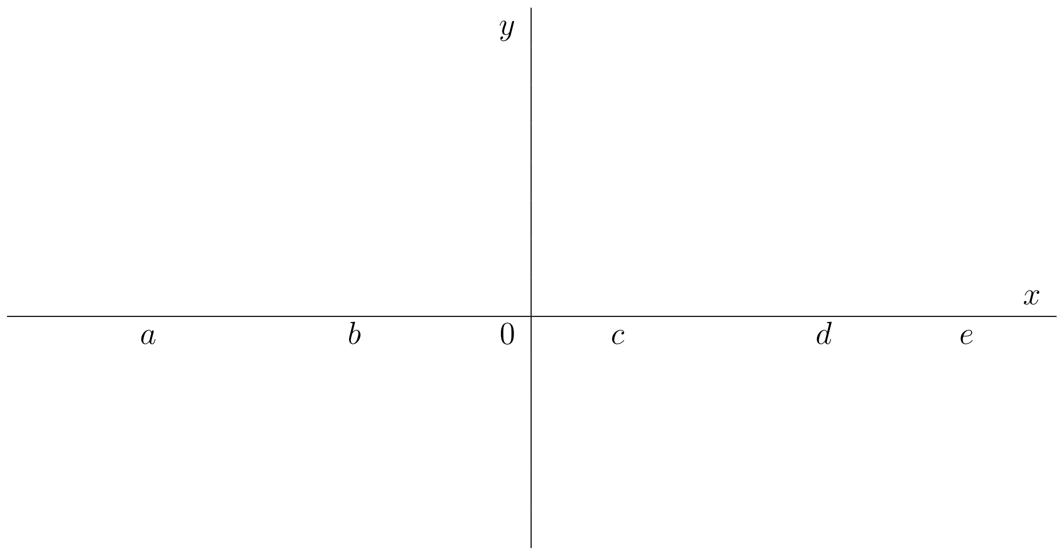


- If $f'(x) > 0$ on an interval, then f is increasing on that interval.
- If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

Let $x = a$ be in the domain of $f(x)$.

- . If the derivative $f'(x)$ goes from positive to negative at $x = a$, then the function f has a *local maximum* at $x = a$.
- If the derivative $f'(x)$ goes from negative to positive at x = a, then the function f has a *local minimum* at $x = a$.

EXAMPLE 1. *Consider the graph of the* **derivative** $f'(x)$ *of some function* f.



Answer the following questions:

(a) *Over what intervals is* f *increasing?* ____________________________

(b) *Over what intervals is* f *decreasing?* ____________________________

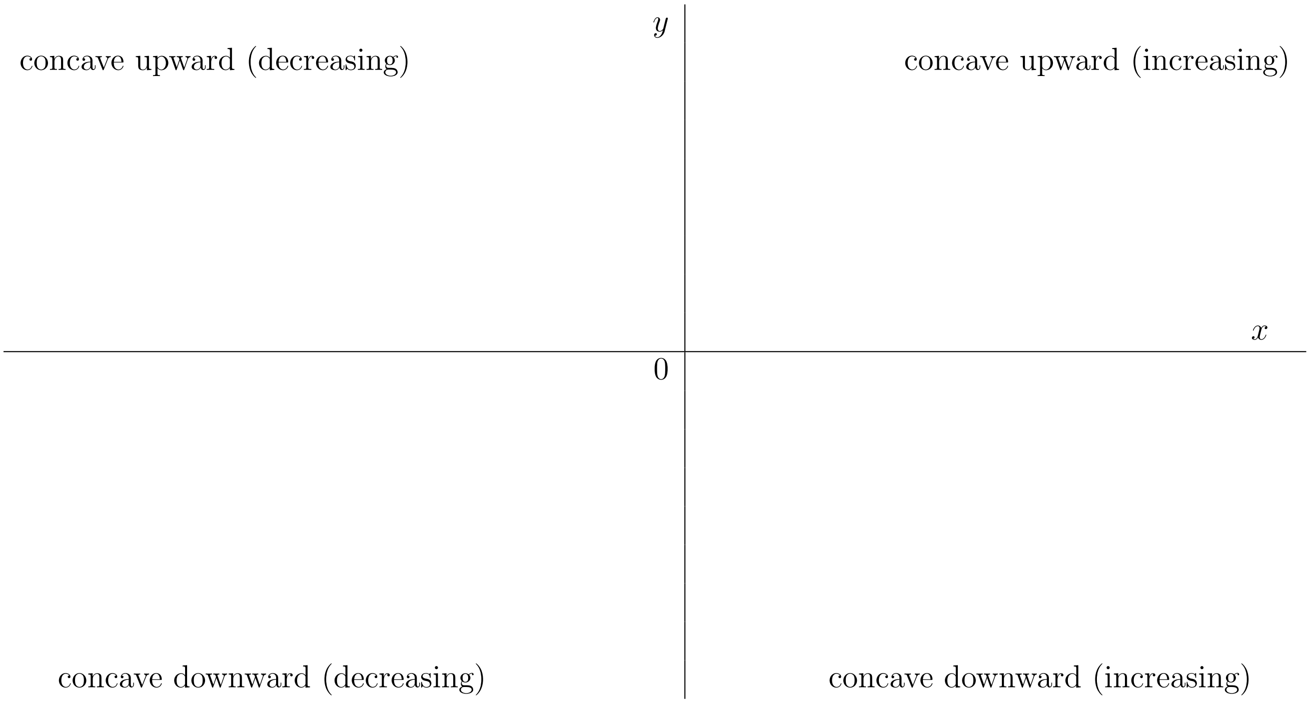
(c) *Determine the* x*-values of* $f(x)$ *that have a horizontal tangent:* ____________________________

(d) *Determine the local maximum point(s) of* f*:* ____________________________

(e) *Determine the local minimum point(s) of* f*:* ____________________________

(f) *Given that* $f(0) = 0$*, sketch a possible graph of* f.

Concavity:

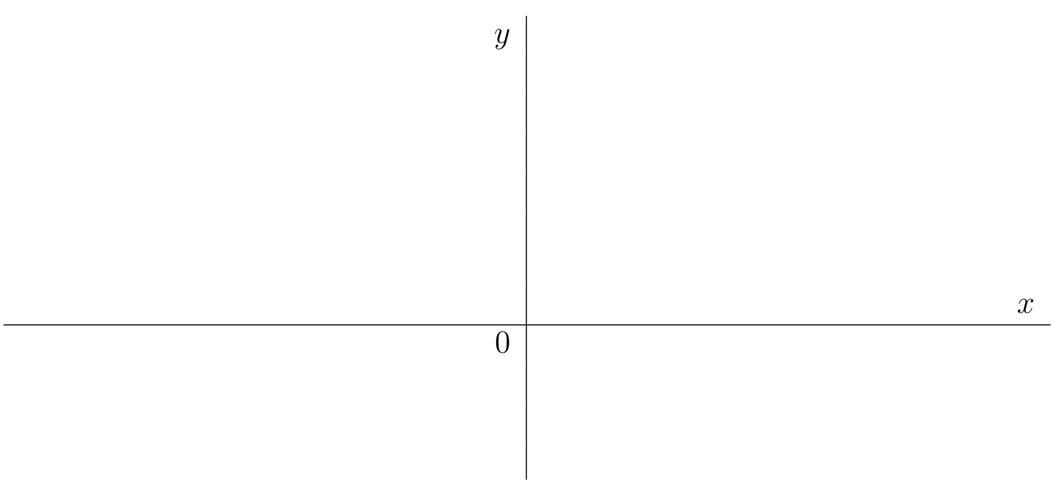


- f is *concave upward* on an interval if all of the tangents to the graph of f on that interval are below the graph of f.
- f is *concave downward* on an interval if all of the tangents to the graph of f on that interval are above the graph of f.

Also,

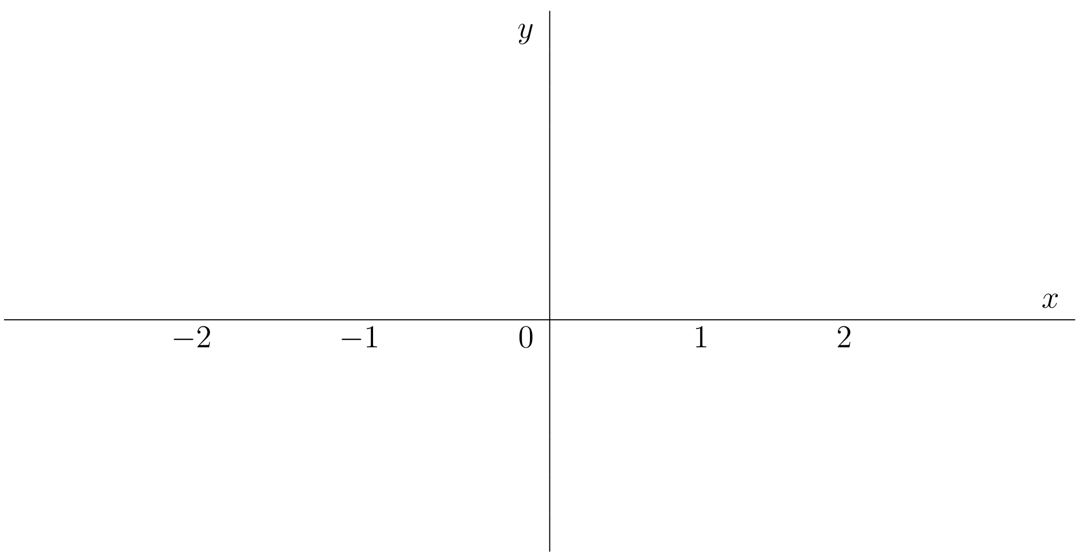
- If the slopes of a curve become progressively larger as x increases, then f is *concave upward.*
- If the slopes of a curve become progressively smaller as x increases, then we say f is *concave downward.*

DEFINITION 2. *If f changes concavity at $x = a$, and $x = a$ is in the domain of f, then $x = a$ is an* **inflection point** *of f.*



EXAMPLE 3. *Sketch a possible graph of a function f that satisfies the following conditions:*

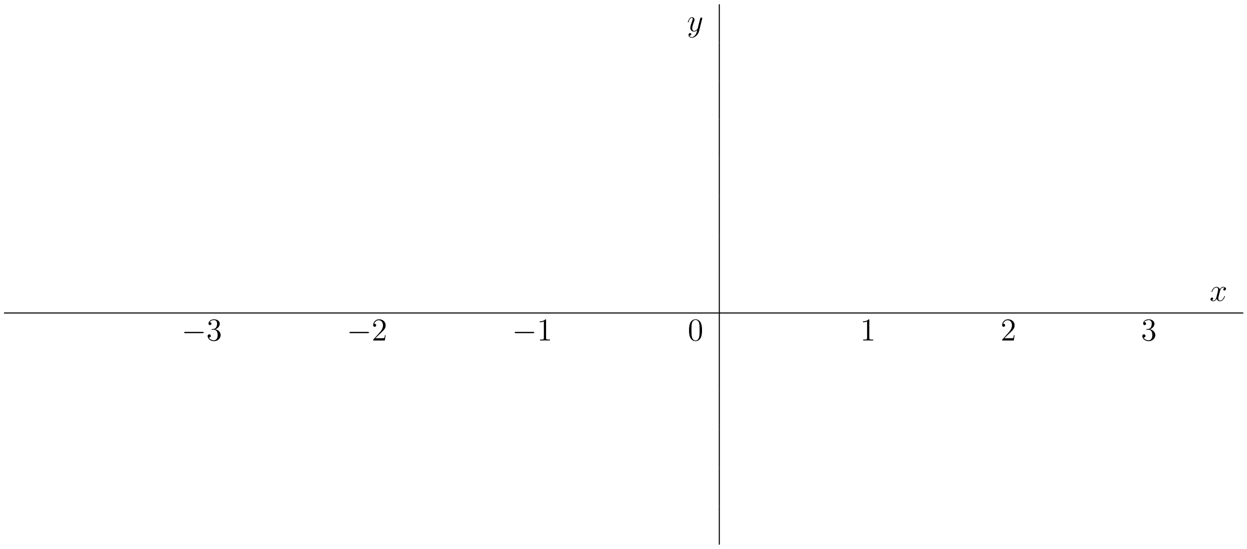
- $f(x)$ *is concave up on* $(-1,0)$ *and* $(1,\infty)$*;*
- $f(x)$ *is concave down on* $(-\infty,1)$ *and* $(0,1)$*;*
- $x = 0, \pm 1$ *are the inflection points;*
- $x = -2$ *is the local maximum point;*
- $x = 2$ *is the local minimum point;*
- $f(0) = 3$.



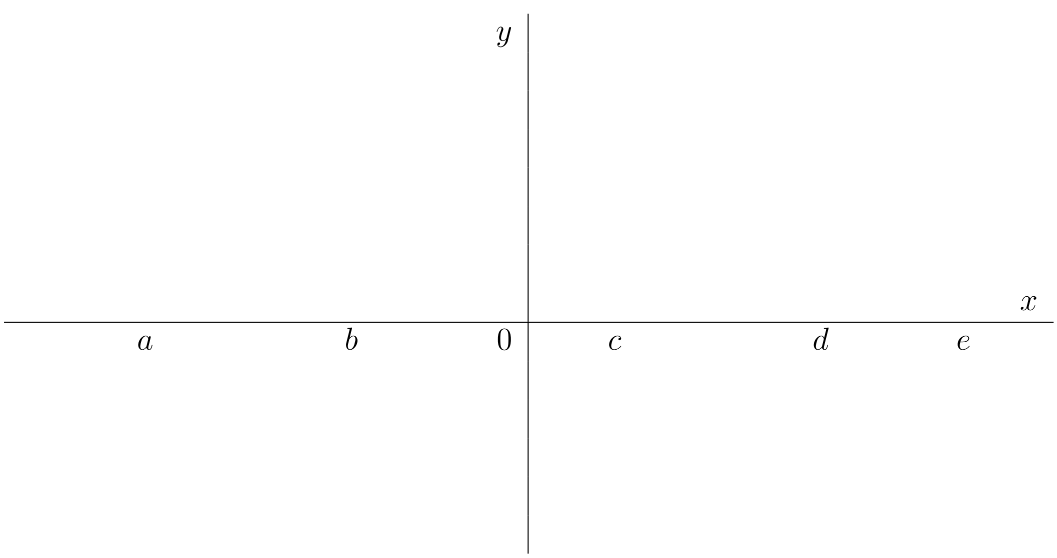
What does f'' say about f?

- If $f''(x) > 0$ for all x on an interval, then f is concave up on that interval.
- If $f''(x) < 0$ for all x on an interval, then f is concave down on that interval.

EXAMPLE 4. *Given the graph of the derivative, $f'(x)$, for some function f. Determine the intervals of concavity and inflection point(s).*



EXAMPLE 5. *Consider the graph of the* **derivative** $f'(x)$ *of some function f.*



Given that $f(0) = 0$, sketch a possible graph of f.