6.1: Sigma notation

DEFINITION 1. *If* $a_m, a_{m+1}, a_{m+2}, ..., a_n$ *are real numbers and* m *and* n *are integers such that* $m \leq n$, *then*

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + \ldots + a_n.$$

EXAMPLE 2. *Compute these summations:*

(a) $\sum_{i=5}^{8} (\sin i - \sin(i+1))$

(b) $\sum_{i=1}^{10} 3$

EXAMPLE 3. *Write the sum in sigma notation:*
$-1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25}$

THEOREM 4. *If* c *is any constant then*

$$\sum_{i=m}^{n} ca_i = c\sum_{i=m}^{n} a_i$$

$$\sum_{i=m}^{n} (a_i + b_i) = \sum_{i=m}^{n} a_i + \sum_{i=m}^{n} b_i$$

$$\sum_{i=m}^{n} (a_i - b_i) = \sum_{i=m}^{n} a_i - \sum_{i=m}^{n} b_i$$

Note that in general

$$\sum_{i=m}^{n} a_i b_i \neq \left(\sum_{i=m}^{n} a_i\right) \cdot \left(\sum_{i=m}^{n} b_i\right).$$

EXAMPLE 5. *If* $\sum_{i=1}^{25} f(i) = 15,\ f(25) = 7$ *and* $\sum_{i=1}^{24} g(i) = 25$ *find*

$$\sum_{i=1}^{24} (2f(i) - g(i))$$

THEOREM 6.

- $\sum_{i=1}^{n} 1 = n$
- $\sum_{i=1}^{n} c = nc$, *where* c *is a constant.*
- $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$
- $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

EXAMPLE 7. *Compute these sums:*

(a) $\sum_{i=4}^{27} 20 =$

(b) $\sum_{i=1}^{n} i(i+4) =$

(c) $\sum_{k=1}^{100} \left(\frac{1}{k} - \frac{1}{k+1}\right) =$

EXAMPLE 8. *Find the limit:* $\lim_{n\to\infty} \sum_{j=1}^{n} \frac{1}{n}\left[\left(\frac{j}{n}\right)^3 + 1\right]$