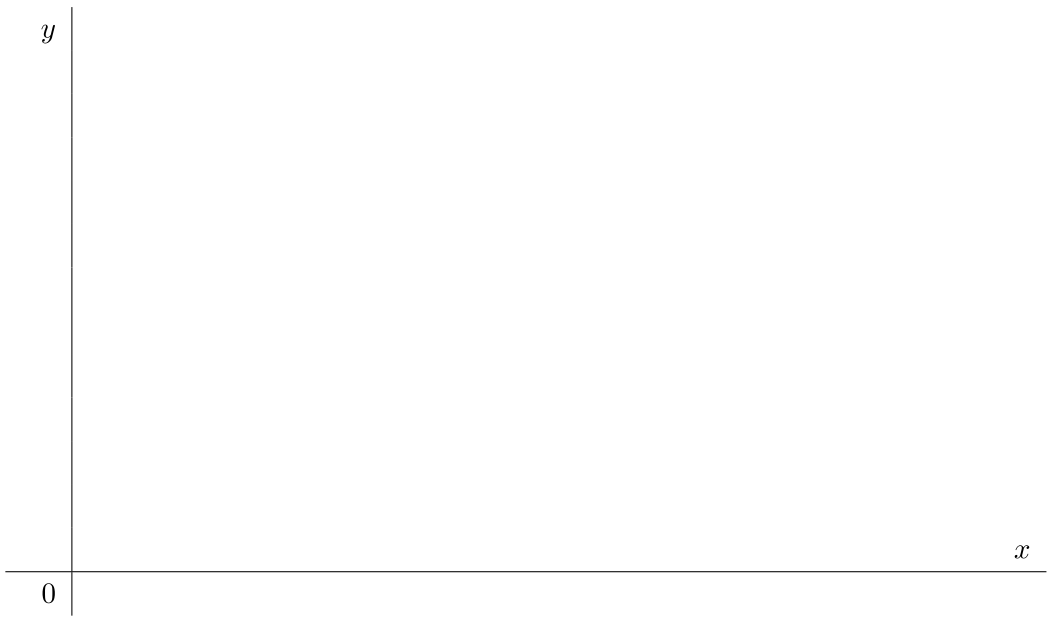


6.2: Area

Area problem: Let a function $f(x)$ be positive on some interval $[a, b]$. Determine the area of the region between the function and the x-axis.



Solution: Choose **partition** points $x_0, x_2, \ldots, x_{n-1}, x_n$ so that

$$a = x_0 \leq x_2 \leq \ldots \leq x_{n-1} \leq x_n = b.$$

Use notation $\Delta x_i = x_i - x_{i-1}$ for the length of ith subinterval $[x_{i-1}, x_i]$ $(1 \leq 1 \leq n)$
The length of the longest subinterval is denoted by $\|P\|$.
The location in each subinterval where we compute the height is denoted by x_i^*.
The area of the ith rectangle is

$$A_i =$$

Then

$$A \approx$$

The area A of the region is:

$$A =$$

EXAMPLE 1. *Given* $f(x) = 100 - x^2$ *on* $[0, 10]$. *Let* $P = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ *and* x_i^* *be left endpoint of* i*th subinterval.*

(a) *Find* $\|P\|$.

(b) *Find the sum of the areas of the approximating rectangles.*

(c) *Sketch the graph of* f *and the approximating rectangles.*

Riemann Sum for a function $f(x)$ on the interval $[a, b]$ is a sum of the form:

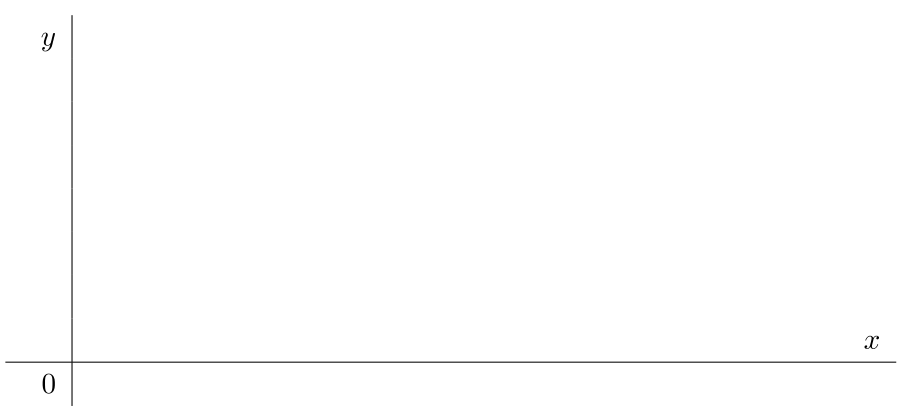
$$\sum_{i=1}^{n} f(x_i^*)\Delta x_i.$$

Consider a partition has equal subintervals: $x_i = a + i\Delta x$, where $\Delta x = \dfrac{b-a}{n}$.

LEFT-HAND RIEMANN SUM : $\quad L_n = \sum_{i=1}^{n} f(x_{i-1})\Delta x = \sum_{i=1}^{n} f(a + (i-1)\Delta x)\Delta x$

RIGHT-HAND RIEMANN SUM : $\quad R_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} f(a+i\Delta x)\Delta x$

MIDPOINT RIEMANN SUM : $\quad M_n = \sum_{i=1}^{n} f\left(\dfrac{x_i + x_{i-1}}{2}\right)\Delta x =$



EXAMPLE 2. *Given* $f(x) = \dfrac{1}{x}$ *on* $[1, 2]$. *Calculate* L_2, R_2, M_2.

EXAMPLE 3. *Represent area bounded by* $f(x)$ *on the given interval using Riemann sum. Do not evaluate the limit.*

(a) $f(x) = x^2 + 2$ *on* $[0, 3]$ *using right endpoints.*

(b) $f(x) = \sqrt{x^2 + 2}$ *on* $[0, 3]$ *using left endpoints.*

EXAMPLE 4. *The following limits represent the area under the graph of $f(x)$ on an interval $[a, b]$. Find $f(x), a, b$.*

(a) $\lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \sqrt{1 + \frac{3i}{n}}$

(b) $\lim_{n\to\infty} \frac{10}{n} \sum_{i=1}^{n} \frac{1}{1 + \left(7 + \frac{10i}{n}\right)^3}$