

Section 3.2: Differentiation formulas

The properties and formulas in this section will be given in both “prime” notation and “fraction” notation.

PROPERTIES:

1. *Constant rule:* If f is a constant function, $f(x) = c$, then $f'(x) = 0$, or $\frac{dc}{dx} = 0$.
2. *Power rule:* If $f(x) = x^n$, where n is a real number, then $f'(x) = nx^{n-1}$, or $\frac{d}{dx}x^n = nx^{n-1}$.
3. *Constant multiple rule:* If c is a constant and $f'(x)$ exists then

$$(cf(x))' = cf'(x), \quad \text{or} \quad \frac{d}{dx}(cf) = c\frac{df}{dx}.$$

4. *Sum/Difference rule:* If $f'(x)$ and $g'(x)$ exists then

$$(f(x) + g(x))' = f'(x) + g'(x), \quad \text{or} \quad \frac{d}{dx}(f + g) = \frac{df}{dx} + \frac{dg}{dx}.$$

5. *Product rule:* If $f'(x)$ and $g'(x)$ exists then

$$(f(x)g(x))' = f'(x)g(x) + f(x)g'(x), \quad \text{or} \quad \frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}.$$

6. *Quotient rule:* If $f'(x)$ and $g'(x)$ exists then

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}, \quad \text{or} \quad \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{[g(x)]^2}.$$

EXAMPLE 1. Find the derivatives of the following functions:

(a) $f(x) = x^{10} + 3x^5 - 12x + 44 - \pi^5$

(b) $g(t) = (1 + \sqrt{t})^2$

(c) $F(s) = \left(\frac{s}{3}\right)^4 - s^{-5}$

$$(d) y = \frac{u^5 + 1}{u^2 \sqrt{u}}$$

$$(e) f(x) = (x^4 - 3x^2 + 11)(3x^3 - 5x^2 + 22)$$

$$(f) g(z) = \frac{4 - z^2}{4 + z^2}$$

EXAMPLE 2. The functions f and g satisfy the properties as shown in the table below:

| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 1 | -5 | 8 | 5 | 12 |
| 3 | 1 | 2 | -2 | 8 |

Find the indicated quantity:

(a) $h'(3)$ if $h(x) = (3x^2 + 1)g(x)$

(b) $H'(1)$ if $H(x) = \frac{x^2}{f(x)}$

EXAMPLE 3. Given $f(x) = x^3 - 5x^2 + 6x - 3$

(a) Find the equation of the tangent line to the graph of $f(x)$ at the point $(1, -1)$.

(b) Find the value(s) of x where $f(x)$ has a tangent line that is parallel to $y = 6x + 1$.

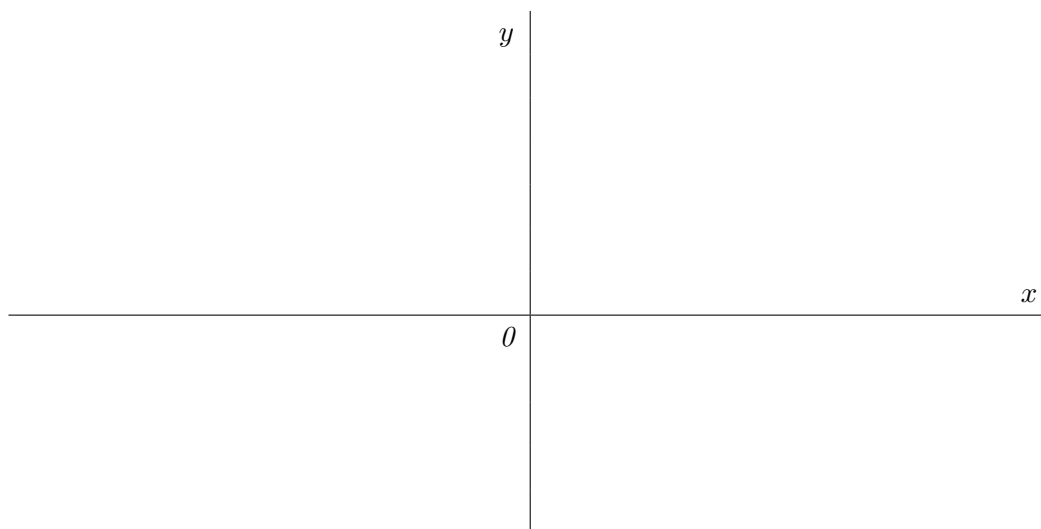
EXAMPLE 4. Show that there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -4)$ and find their equations.

EXAMPLE 5. Let $f(x) = \begin{cases} -1 - 2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 1 \\ x & \text{if } x > 1 \end{cases}$

(a) Give a formula for f' .

(b) For what value(s) of x the function is not differentiable?

(c) Sketch the graph of f and f' on the same axis.



EXAMPLE 6. A ball is thrown into the air. Its position at time t is given by

$$\mathbf{s}(t) = \langle 2t, 10t - t^2 \rangle.$$

(a) Find the velocity of the ball at time $t = 2$.

(b) Find the speed of the ball at time $t = 2$.

EXAMPLE 7. Let

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ ax + b & \text{if } x > 2 \end{cases}$$

Find the values of a and b that makes f differentiable everywhere.