Section 3.5: Chain Rule

Question: `How to find the derivatives of the following functions:`

$$y = (x^6 + 4x^2 + 12)^{15}; \qquad y = \sec(12x^2) + \tan^3(x) \qquad y = \sqrt[3]{4+x}$$

`Review of Composite Functions:`

$$[f \circ g](x) = f(g(x))$$

If $f(x) = x^{15}$ and $g(x) = x^6 + 4x^2 + 12$ then $[f \circ g](x) =$

Conversely, if $[f \circ g](x) = \sec(12x^2)$ then $f(x) =$ and $g(x) =$

The CHAIN RULE: *If the derivatives $g'(x)$ and $f'(x)$ both exist, and $F = f \circ g$ is the composite defined by*

$$F(x) = f(g(x))$$

then

$$F'(x) = f'(g(x))g'(x).$$

In Leibniz notation: If the derivatives of $y = f(u)$ and $u = g(x)$ both exist then

$$y = f(g(x))$$

is differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

$y = f(x)$	$u(x)$	$f(u)$	$\frac{dy}{dx}$
$y =$ $(x^6 + 4x^2 + 12)^{15}$	$u =$ $u' =$	$y =$ $y' =$	$\frac{dy}{dx} =$
$y = \sec(12x^2)$	$u =$ $u' =$	$y =$ $y' =$	$\frac{dy}{dx} =$
$y = \tan^3(x)$	$u =$ $u' =$	$y =$ $y' =$	$\frac{dy}{dx} =$
$y = \sqrt[3]{4+x}$	$u =$ $u' =$	$y =$ $y' =$	$\frac{dy}{dx} =$
$y = [g(x)]^n$	$u =$ $u' =$	$y =$ $y' =$	$\frac{dy}{dx} =$ Generalized Power Rule

EXAMPLE 1. *Find the derivative:*

(a) $f(x) = \dfrac{1}{(x^3 + 5x^2 + 12)^{2012}}$

(b) $h(x) = x^8(3\sqrt{x} - 11)^8$

(c) $f(x) = \cos(5x) + \cos^5 x$

(d) $f(x) = \sqrt{x^3 + \sqrt{x^2 + \sqrt{x}}}$

EXAMPLE 2. *Find F' and G' if*

$$F(x) = f(\sin x), \qquad G(x) = \sin(f(x)),$$

where $f(x)$ is a differentiable function.

EXAMPLE 3. *Let $f(x)$ and $g(x)$ be given differentiable functions satisfy the properties as shown in the table below:*

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-5	8	3	12
3	1	2	-2	8

Suppose that $h = f \circ g$. Find $h'(1)$.