3.7: Derivatives of the vector functions

DEFINITION 1. *If* $\mathbf{r}(t)$ *is a vector function then the* **tangent vector v** *at* $t = a$ *is found by*

$$\mathbf{v} = \lim_{t \to a} \frac{1}{t-a} \left[\mathbf{r}(t) - \mathbf{r}(a) \right].$$

DEFINITION 2. *The* **derivative of a vector function** $\mathbf{r}(t)$ *at a number* a*, denoted by* $\mathbf{r}'(t)$*, is*

$$\mathbf{r}'(t) = \lim_{t \to a} \frac{1}{t-a} \left[\mathbf{r}(t) - \mathbf{r}(a) \right],$$

if this limit exists.

One can show that this definition implies that

If $(r)(t) = \langle x(t), y(t) \rangle$ *is a vector function, then*

$$\mathbf{r}'(t) = \left\langle x'(t), y'(t) \right\rangle$$

if both $x'(t), y'(t)$ *exist.*

EXAMPLE 3. *If* $\mathbf{r}(\mathbf{t}) = \left\langle t^2, \sqrt{t-5} \right\rangle$ *find the domain of* $\mathbf{r}(t)$ *and* $\mathbf{r}'(t)$*.*

EXAMPLE 4. *Given curve* $\mathbf{r}(t) = \left\langle 2t, 10t - t^2 \right\rangle$*.*

(a) *Find a vector tangent to the curve at the point* $(4, 16)$*.*

(b) *Find parametric equations of the tangent line to* $\mathbf{r}(t)$ *at* $t = 2$.

(c) *Find a Cartesian equation of this tangent line.*

DEFINITION 5. *If* $(r)(t) = \langle x(t), y(t) \rangle$ *is a vector function representing the position of a particle at time* t*, then*

- **instantaneous velocity** *at time* t *is* $\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle$
- **instantaneous speed** *at time* t *is* $|\mathbf{r}'(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$

EXAMPLE 6. *The vector function* $\mathbf{r}(t) = \left\langle t, \sqrt{t^2 + 9} \right\rangle$ *represents the position of a particle at time* t*. Find the velocity and speed of the particle at time* $t = 4$.