Fall 2012 Math 152

Week in Review 1

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(covering Sections 6.5&7.1)

6.5: The substitution rule

Key Points

If $u = g(x)$ is a differentiable function, then

$$\int f(g(x))g'(x)\,\mathrm{d}x = \int f(u)\,\mathrm{d}u$$

$$\int_a^b f(g(x))g'(x)\,\mathrm{d}x = \int_{g(a)}^{g(b)} f(u)\,\mathrm{d}u$$

You should make sure that the old variable x has disappeared from the integral.

Examples

Evaluate the following integrals

1. $\displaystyle\int e^{2012x}\,\mathrm{d}x$
2. $\displaystyle\int_2^4 \sin(4\pi x)\,\mathrm{d}x$
3. $\displaystyle\int_0^{\pi/2} \cos^7 x \sin x\,\mathrm{d}x$
4. $\displaystyle\int_0^1 x^4 e^{9x^5-8}\,\mathrm{d}x$
5. $\displaystyle\int \frac{x^{10}}{x^{11}+11}\,\mathrm{d}x$
6. $\displaystyle\int \frac{3}{\sqrt{3y+1}}\,\mathrm{d}y$
7. $\displaystyle\int \frac{\tan(\ln x)}{x}\,\mathrm{d}x$
8. $\displaystyle\int \frac{e^x}{\sqrt{1-e^{2x}}}\,\mathrm{d}x$

9. $\int x^5 \sqrt{4+x^3}\,\mathrm{d}x$

10. $\int \frac{x-1}{x^2+1}\,\mathrm{d}x$

7.1: Area Between Curves

Key Points

CASE I $A = \int_a^b \left(\begin{array}{c}\text{upper}\\ \text{function}\end{array}\right) - \left(\begin{array}{c}\text{lower}\\ \text{function}\end{array}\right) \mathrm{d}x$

CASE II $A = \int_c^d \left(\begin{array}{c}\text{right}\\ \text{function}\end{array}\right) - \left(\begin{array}{c}\text{left}\\ \text{function}\end{array}\right) \mathrm{d}y$

- In some cases the limits of integration can be determined as the intersection points of two curves.
- Sketch of a graph of the region is highly recommended.
- The area between two curves will always be positive.

Examples

11. Find the area of the region bounded by $y = \sin x$, $y = 0$, $x = \pi/4$, $x = \pi/2$.
12. Find the area of the region

$$D = \{(x, y) : \pi/4 \le x \le 3\pi/2, 0 \le y \le \sin x\}.$$

13. Find the area of the region bounded by $y = x^3$ and $y^2 = x$.
14. Determine the area of the region enclosed by $x = -y^2 + 10$, $x = (y-2)^2$.
15. Find the area of the region bounded by $y = x^2 - 3$ and $y = \frac{5}{1+x^2}$.
16. Find the area of the region bounded by $y = \sin 2x$, $y = \sin 4x$, $x = \pi/8$, and $x = \pi/4$. Do not evaluate. Just set up the integral.
17. Determine the area of the region bounded by the x-axis, the curve $y = x^2$ and tangent line to this curve at the point $(1, 1)$.