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Fall 2012 Math 152

Week in Review 11

courtesy: *Oksana Shatalov*

(covering Sections 11.1-11.2 & Sample Test 3)

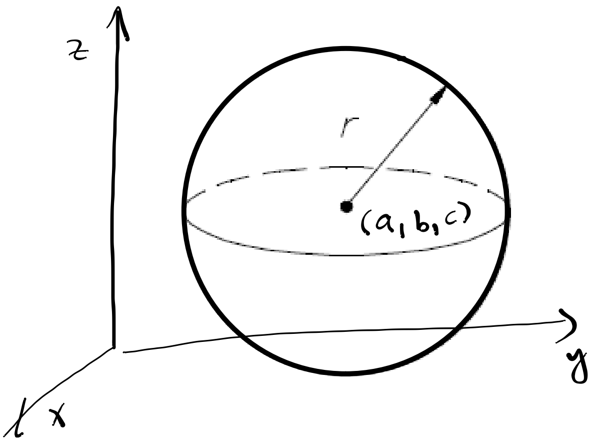
11.1: Three-dimensional Coordinate System

Key Points

- The distance between the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is

$$|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

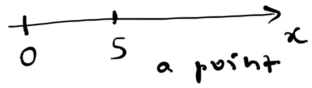
- Equation of a sphere $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ (completing the square)



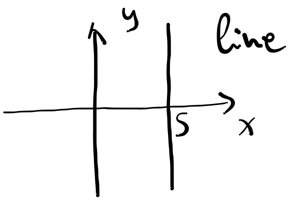
Examples

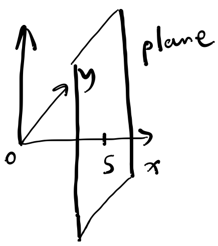
1. Graph the following regions:
   (a) $x = 5$ in $\mathbb{R}, \mathbb{R}^2, \mathbb{R}^3$; **(b)** $x^2 + y^2 - 1 = 0$ in $\mathbb{R}^2, \mathbb{R}^3$.

$\mathbb{R}$

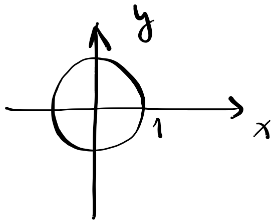


$\mathbb{R}^2$

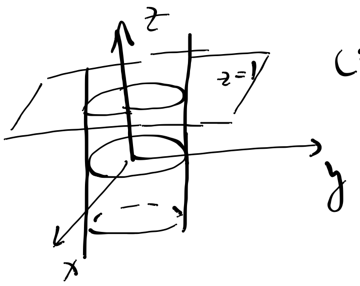




$\mathbb{R}^2$



$\mathbb{R}^3$



circular cylinder

2. Given the sphere $(x-1)^2 + (y+4)^2 + (z-2)^2 = 16$.

   (a) What is the intersection of the sphere with the yz-plane.

   (b) Find the distance from the point $(1, -2, 3)$ to the center of the sphere.

(a) yz-plane $\iff \begin{cases} x = 0 \\ (x-1)^2 + (y+4)^4 + (z-2)^2 = 16 \end{cases}$

$(-1)^2 + (y+4)^2 + (z-2)^2 = 16$

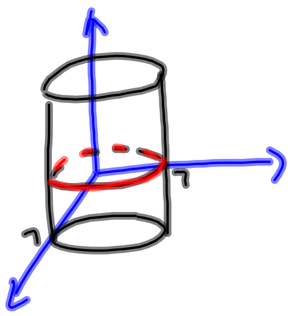
$\boxed{(y+4)^2 + (z-2)^2 = 15, \quad x = 0}$

circle

(b) distance $\left((1,-2,3), (1,-4,2) \right)$

$d = \sqrt{(1-1)^2 + (-2-(-4))^2 + (3-2)^2} = \sqrt{0^2 + 2^2 + 1^2} = \sqrt{5}$

3. What is the intersection of the surface $\underbrace{x^2 + y^2 = 49}_{\text{cylinder}}$ with the $\underbrace{xy\text{-plane}}_{z=0}$.



circle centered at origin with radius $\sqrt{49} = 7$

4. Determine the radius and the center of the sphere given by the equation

$$x^2 + y^2 + z^2 + 2y + z - 1 = 0.$$

Completing square

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

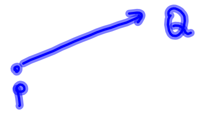
$$x^2 + \underbrace{y^2 + \overset{2\cdot y\cdot 1}{2y} + 1^2}_{(y+1)^2} - 1^2 + \underbrace{z^2 + \overset{2\cdot z\cdot\frac{1}{2}}{z} + \left(\tfrac{1}{2}\right)^2}_{\left(z+\frac{1}{2}\right)^2} - \left(\tfrac{1}{2}\right)^2 = 1$$

$$x^2 + (y+1)^2 + \left(z+\tfrac{1}{2}\right)^2 = 1 + 1 + \tfrac{1}{4} = \tfrac{9}{4}$$

center $\left(0, -1, -\frac{1}{2}\right)$, $r = \sqrt{\frac{9}{4}} = \frac{3}{2}$

11.2: Vectors and the Dot Product in Three Dimensions

Key Points

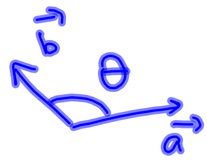


- The vector $\mathbf{a}$ from the point $P(x_1, y_1, z_1)$ to $Q(x_2, y_2, z_2)$ is

$$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}.$$

- **The magnitude or length** of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

- `Unit vector:` $\hat{\mathbf{a}} = \dfrac{\mathbf{a}}{|\mathbf{a}|}$



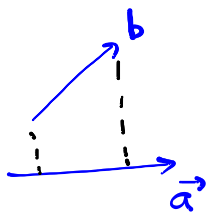
- **Dot Product:**

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \cdot |\mathbf{b}| \cos\theta = a_1 b_1 + a_2 b_2 + a_3 b_3,$$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, $0 \le \theta \le \pi$.

- Scalar projection of vector $\mathbf{b}$ onto vector $\mathbf{a}$: $\texttt{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$

- Vector projection of vector $\mathbf{b}$ onto vector $\mathbf{a}$: $\texttt{proj}_{\mathbf{a}}\mathbf{b} = \underbrace{\left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}\right)}_{\text{comp}_{\vec a}\vec b} \overbrace{\left(\dfrac{\mathbf{a}}{|\mathbf{a}|}\right)}^{\text{unit } \hat a} = \left(\dfrac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$



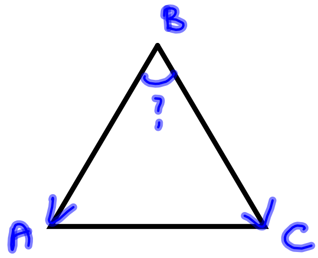
Examples

5. Given the triangle with vertices $A(2,4,5)$, $B(3,5,3)$, and $C(2,8,-3)$.

   (a) Find the cosine of the angle at B.

   (b) Compute $\text{comp}_{\vec{AB}}\vec{BC}$.

   (c) Compute $\text{proj}_{\vec{AB}}\vec{BC}$.



(a) $\cos\hat{B} = \dfrac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}|\cdot|\vec{BC}|}$

$\vec{BA} = \langle 2,4,5\rangle - \langle 3,5,3\rangle = \langle -1,-1,2\rangle$

$\vec{BC} = \langle 2,8,-3\rangle - \langle 3,5,3\rangle = \langle -1,3,-6\rangle$

$\vec{BA}\cdot\vec{BC} = \langle -1,-1,2\rangle\cdot\langle -1,3,-6\rangle = -1\cdot(-1) + (-1)\cdot 3 + 2\cdot(-6)$

$= 1 - 3 - 12 = \boxed{-14}$

$|\vec{BA}| = \sqrt{(-1)^2 + (-1)^2 + 2^2} = \sqrt{6}$

$|\vec{BC}| = \sqrt{(-1)^2 + 3^2 + (-6)^2} = \sqrt{1+9+36} = \sqrt{46}$

$\cos\hat{B} = \dfrac{-14}{\sqrt{6}\cdot\sqrt{46}}$

(b)

$$\text{comp}_{\vec{AB}}\vec{BC} = \frac{\vec{AB}\cdot\vec{BC}}{|\vec{AB}|}$$

$$\vec{AB} = -\vec{BA}$$

(a)

$$\vec{AB}\cdot\vec{BC} = -\vec{BA}\cdot\vec{BC} = -(-14) = 14$$

$$|\vec{AB}| = |\vec{BA}| = \sqrt{6}$$

$$\text{comp}_{\vec{AB}}\vec{BC} = \frac{14}{\sqrt{6}}$$

(c) $$\text{proj}_{\vec{AB}}\vec{BC} = \left(\text{comp}_{\vec{AB}}\vec{BC}\right)\cdot\frac{\vec{AB}}{|\vec{AB}|}$$

(a)+(b)

$$= \frac{14}{\sqrt{6}}\ \frac{-\langle -1,-1,2\rangle}{\sqrt{6}}$$

$$= \frac{14}{6}\langle 1,1,-2\rangle = \frac{7}{3}\langle 1,1,-2\rangle$$

$$= \left\langle \frac{7}{3}, \frac{7}{3}, -\frac{14}{3}\right\rangle$$

OR $$\frac{7}{3}\vec{i} + \frac{7}{3}\vec{j} - \frac{14}{3}\vec{k}$$

6. Find a unit vector in the direction $\mathbf{b} - \mathbf{a}$ where $\mathbf{a} = \langle 3, -7, 0\rangle$ and $\mathbf{b} = \langle 1, -6, -2\rangle$.

$$\vec{b} - \vec{a} = \langle 1, -6, -2\rangle - \langle 3, -7, 0\rangle =$$

$$\langle 1-3, -6-(-7), -2-0\rangle =$$

$$\langle -2, 1, -2\rangle$$

$$|\vec{b} - \vec{a}| = |\langle -2, 1, -2\rangle| =$$

$$\sqrt{(-2)^2 + 1^2 + (-2)^2} = 3$$

$$\widehat{b-a} = \frac{\vec{b} - \vec{a}}{|\vec{b} - \vec{a}|} = \frac{\langle -2, 1, -2\rangle}{3} =$$

$$\langle -\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\rangle$$

Sample Test 3

8. Which of the following series converges absolutely?

NO (a) $\sum_{n=1}^{\infty}(-1)^{n+5}$ $\quad \lim_{n\to\infty}(-1)^{n+5}$ DNE $\Rightarrow$ the series Diverges · by DT

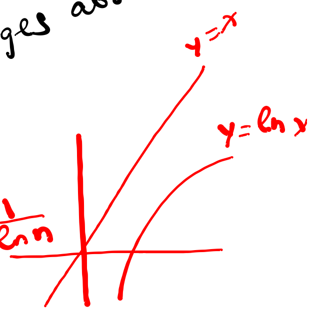
YES (b) $\sum_{n=1}^{\infty}\frac{\sin(\pi^3 n^2)}{n^2\sqrt{n}}$; $\left|\frac{\sin \pi^3 n^2}{n\sqrt{n}}\right| < \frac{1}{n^{5/2}}$) by Comp. Test the series of abs. values converges

$\sum \frac{1}{n^{5/2}}$ convergent $\Rightarrow$ the given series converges absolutely

NO (c) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[4]{n}}$ b/c $\sum \frac{1}{\sqrt[4]{n}}$ diverges, $p = \frac{1}{4} < 1$

NO (d) $\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}$

$\frac{1}{\ln n} > \frac{1}{n}$ $\quad$ $\sum \frac{1}{n}$ divergent $\Big\}$ by Comp. Test the series $\sum \frac{1}{\ln n}$ diverges



$0 < \ln x < x$

$x \geq 2$ $\quad \frac{1}{\ln x} > \frac{1}{x}$

YES (e) $\sum_{n=1}^{\infty}\frac{n^n}{(n!)^2}$

Ratio Test

$L = \lim_{n\to\infty}|a_{n+1}|\cdot\frac{1}{|a_n|} = \lim_{n\to\infty}\frac{(n+1)^{n+1}}{(n+1)!^2}\cdot\frac{n!^2}{n^n}$

$= \lim_{n\to\infty}\frac{(n+1)^n\cdot\cancel{(n+1)}}{\cancel{n!^2}(n+1)^{\cancel{2}}}\cdot\frac{\cancel{n!^2}}{n^n} = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n\cdot\frac{1}{n+1} = 0 < 1$

the series conv. absolutely

NO (f) $\sum_{n=1}^{\infty}\frac{5^n}{\ln(n+1)}$

$\lim_{n\to\infty}\frac{5^n}{\ln(n+1)} = \frac{\infty}{\infty}$ L'Hospital's

$= \lim_{n\to\infty}\frac{5^n \ln 5}{\frac{1}{n+1}}$

$= \ln 5 \lim_{n\to\infty} 5^n(n+1) = \infty$

The series diverges by DT.

$R=4$

9. Suppose that the power series $\sum_{n=1}^{\infty} c_n(x-4)^n$ has the radius of convergence 4. Consider the following pair of series:

$$\boxed{(I)\quad \sum_{n=1}^{\infty} c_n 5^n} \qquad (II)\quad \sum_{n=1}^{\infty} c_n 3^n.$$

$x-4=5 \Rightarrow x=9$
divergent

$x-4=3 \Rightarrow x=7$
converges

Which of the following statements is true?

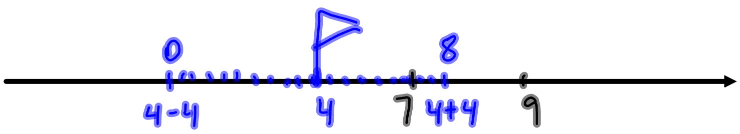
(a) (I) is convergent, (II)is divergent

(b) Neither series is convergent

(c) Bith series are convergent

(d) (I) is divergent, (II) is convergent

(e) no conclusion can be drawn about either series.



10. Show that the series $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ converges. Then find un upper bound on the error in using s_{10} to approximate the series. (Note that $\ln 2 > 1/2$.)

Show that $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ converges

Let's see if $f(x) = \frac{\ln x}{x^2}$ is easy to integrate

$u = \ln x$; $dv = x^{-2} dx$

$du = \frac{dx}{x}$; $v = -x^{-1}$

$$\int \frac{\ln x}{x^2} dx = \int x^{-2} \ln x \, dx$$

$$= -x^{-1} \ln x - \int -x^{-1} \frac{dx}{x} = -\frac{\ln x}{x} + \int \frac{dx}{x^2} = -\frac{\ln x}{x} - \frac{1}{x}$$

$$= -\left[\frac{\ln x}{x} + \frac{1}{x}\right]$$

$f(x) = \frac{\ln x}{x^2}$ is positive, continuous on $[2, \infty)$

Is $f(x)$ decreasing? $f'(x) = \frac{\frac{1}{x} \cdot x^2 - 2x \ln x}{x^4}$

$$= \frac{x(1 - 2\ln x)}{x^4} < 0$$

decreasing.

By Integral Test, the series conv./diverges together with $\int_2^{\infty} f(x) dx = \lim_{t \to \infty} \int_2^t \frac{\ln x}{x^2} =$

$$= \lim_{t \to \infty} -\left[\frac{\ln x}{x} + \frac{1}{x}\right]\Big|_2^t$$

$$= -\lim_{t \to \infty} \underbrace{\frac{\ln t}{t} + \frac{1}{t}}_{\to 0 \text{ (use L'Hospitals Rule)}} - \frac{\ln 2}{2} - \frac{1}{2}$$

$$= \frac{\ln 2}{2} + \frac{1}{2}$$

The series converges

Find an upper bound for R_{10}

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^2} \simeq S_{10}$$

By Integral Test Remainder estimate :

$$R_n \leq \int_n^{\infty} f(x)\,dx$$

$$R_{10} \leq \int_{10}^{\infty} \frac{\ln x}{x^2}\,dx = \frac{\ln 10}{10} + \frac{1}{10}$$

$$\simeq 0.33$$

Maclaurin series

11. If we represent $\frac{x^2}{4+9x^2}$ as a power series centered at $a = 0$, what is the associated radius of convergence?

$$f(x) = \frac{x^2}{4+9x^2} = x^2 \cdot \frac{1}{4+9x^2}$$

$$= x^2 \cdot \frac{1}{4} \cdot \frac{1}{1+\frac{9x^2}{4}} = \frac{x^2}{4} \cdot \underbrace{\frac{1}{1-\left(-\frac{9x^2}{4}\right)}}_{\text{sum of geometric series with common ratio}}$$

$$|r| = \left|-\frac{9x^2}{4}\right| < 1$$

$$-1 < \frac{9x^2}{4} < 1$$

$\sqrt{x^2} = |x|$

$$-\frac{4}{9} < x^2 < \frac{4}{9}$$

$$|x| < \frac{2}{3} \Rightarrow \boxed{R = \frac{2}{3}}$$

12. Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-2)^n(3x-1)^n}{n}$.

$$|a_n| = \frac{2^n |3x-1|^n}{n}$$

$$|a_{n+1}| = \frac{2^{n+1} |3x-1|^{n+1}}{n+1}$$

Apply Ratio Test

$$L = \lim_{n\to\infty} |a_{n+1}| \cdot \frac{1}{|a_n|}$$

$$= \lim_{n\to\infty} \frac{2^{\cancel{n+1}} |3x-1|^{\cancel{n+1}}}{n+1} \cdot \frac{n}{\cancel{2^n} \cancel{|3x-1|^n}} =$$

$$2|3x-1| \lim_{n\to\infty} \frac{n}{n+1} = \underbrace{2|3x-1| < 1}$$

$$|3x-1| < \frac{1}{2}$$

$$|x - \tfrac{1}{3}| < \frac{1}{6}$$

center radius

$$\boxed{R = \frac{1}{6}}$$

Interval of converges
(L = 1) Test endpoints

$$\Downarrow$$

$$2|3x-1| = 1$$

$$3x-1 = \pm\frac{1}{2}$$

$$3x-1 = -\frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{(-2)^n \cdot (-\frac{1}{2})^n}{n} =$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ divergent}$$

$$3x-1 = \frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{(-2)^n (\frac{1}{2})^n}{n} =$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

converges by AST

$$-\frac{1}{2} < 3x-1 \leq \frac{1}{2}$$

$$-\frac{1}{2}+1 < 3x-1+1 \leq \frac{1}{2}+1$$

$$\frac{1}{2} < 3x \leq \frac{3}{2}$$

$$\boxed{\frac{1}{6} < x \leq \frac{1}{2}}$$ or $\left(\frac{1}{6}; \frac{1}{2}\right]$

13. Which of the following statements is TRUE?

F (a) If $a_n > 0$ for $n \geq 1$ and $\sum_{n=1}^{\infty} (-1)^n a_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges.

T (b) If $a_n > 0$ for $n \geq 1$ and $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

F (c) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges. (a_n is not positive or $\{a_n\}$ is not decreasing)

F (d) If $a_n > 0$ for $n \geq 1$ and $\underbrace{\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{e}{2}}_{\text{Ratio Test}}$ then $\sum_{n=1}^{\infty} a_n$ converges. $e \approx 2.7$, $\frac{e}{2} > 1 \Rightarrow \sum a_n$ diverges

(a) $a_n = \frac{1}{n} > 0$

$\sum \frac{(-1)^n}{n}$ converges

but $\sum \frac{1}{n}$ diverges

(b) $\sum a_n$ conv. $\Rightarrow \sum (-1)^n a_n$ converges absolutely $\Rightarrow$ converges.

14. Find a Maclaurin series representation for $\dfrac{e^x - 1 - x}{x^2}$.

We know

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \sum_{n=2}^{\infty} \frac{x^n}{n!}$$

$$\frac{e^x - 1 - x}{x^2} = \frac{1}{x^2}\left(\cancel{1+x} + \sum_{n=2}^{\infty} \frac{x^n}{n!} \cancel{-1-x}\right)$$

$$= \frac{1}{x^2} \sum_{n=2}^{\infty} \frac{x^n}{n!} = \boxed{\sum_{n=2}^{\infty} \frac{x^{n-2}}{n!}}$$

15. (a) Find a Maclaurin series representation for $f(x) = \sin\left(\frac{x^2}{4}\right)$

We know that

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad (-\infty, \infty)$$

$$\sin\left(\frac{x^2}{4}\right) = \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x^2}{4}\right)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{4^{2n+1}(2n+1)!}$$

(b) Write $\int_0^1 \sin\left(\frac{x^2}{4}\right) dx$ as an infinite series.

$$\int_0^1 \sin\left(\frac{x^2}{4}\right) dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n+1}(2n+1)!} \int_0^1 x^{4n+2}\, dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n+1}(2n+1)!} \left.\frac{x^{4n+3}}{4n+3}\right|_0^1 = \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n+1}(2n+1)!\,(4n+3)}}_{\text{alternating Series}}$$

(c) Using the series found in the previous part, find s_2, the second partial sum of the series and give an upper bound on the error $|s - s_2|$.

$$|s - s_2| = |R_2|$$

$$\int_0^1 \sin\frac{x^2}{4}\, dx = \underbrace{\frac{1}{4\cdot 3} - \frac{1}{4^3\cdot 3!\, 7}}_{s_2} + \frac{1}{4^5\cdot 5!\, 11} - \cdots\cdots$$

bound for R_2

$$|s - s_2| = |R_2| \leq \frac{1}{4^5 \cdot 5!\, 11}$$

16. Let $f(x) = e^{5-x}$.

$T_4(x)$

(a) Give the fourth degree Taylor polynomial for $f(x)$ centered around $a = 5$.

$$T_4(x) = f(5) + f'(5)(x-5) + \frac{f''(5)}{2!}(x-5)^2 + \frac{f'''(5)}{3!}(x-5)^3 + \frac{f^{(4)}(5)}{4!}(x-5)^4$$

$f(x) = e^{5-x} = f''(x) = f^{(4)}(x)$ | $a=5$ | $f(5) = f''(5) = f^{(4)}(5) = 1$

$f'(x) = -e^{5-x} = f'''(x)$ | $f'(5) = f'''(5) = -1$

$$T_4(x) = 1 - (x-5) + \frac{1}{2}(x-5)^2 - \frac{1}{6}(x-5)^3 + \frac{1}{24}(x-5)^4$$

(b) Use Taylor's inequality to give a bound on the error when using the fourth degree Taylor polynomial for $f(x)$ to estimate $f(x)$ on the interval [3, 6].

$f(x) = T_4(x) + \underbrace{R_4(x)}$

$f(x) \approx T_4(x)$

Taylor's Inequality:

$$|R_N(x)| \le \frac{M}{(N+1)!}|x-a|^{N+1}$$

where $\left|f^{(N+1)}(x)\right| \le M$ for all x in an interval containing a.

$N=4$, $f(x) = e^{5-x}$, $a=5$, interval $[3,6]$

$$|R_4(x)| \le \frac{M}{(4+1)!}|x-5|^{4+1}, \text{ where } M = \max_{[3,6]}\left|f^{(5)}(x)\right|$$

$$|R_4(x)| \le \frac{M}{5!}|x-5|^5 \le \frac{M}{120}2^5 = \frac{32}{120}M = \frac{4M}{15} \; ☆$$

$3 \le x \le 6$

$3-5 \le x-5 \le 6-5$

$-2 \le x-5 \le 1$

$1 \le |x-5| \le 2$

Determine M: by part (a)

$$\left|f^{(5)}(x)\right| = \left|-e^{5-x}\right| = e^{5-x} = e^5 \cdot \underbrace{e^{-x}}$$

positive decreasing function then abs. maximum is attend at the left end point of the given interval, i.e. $x=3$

$$M = \left|f^{(5)}(3)\right| = e^{5-3} = e^2$$

$$|R_4(x)| \le \frac{4M}{15} = \boxed{\frac{4e^2}{15}}$$

17. Find a Maclaurin series of $f(x) = \ln(2 - x)$ and the associated radius of convergence.

$$f'(x) = -\frac{1}{2-x} = -\frac{1}{2}\cdot\frac{1}{1-\frac{x}{2}} = -\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^n$$

$$= -\sum_{n=0}^{\infty}\frac{x^n}{2^{n+1}}$$

sum of geom. series with common ratio

$-1 < r = \frac{x}{2} < 1$

$-2 < x < 2 \Rightarrow \boxed{R = 2}$

$$f(x) = \int f'(x)\,dx = -\sum_{n=0}^{\infty}\frac{1}{2^{n+1}}\int x^n\,dx = -\sum_{n=0}^{\infty}\frac{x^{n+1}}{2^{n+1}(n+1)} + C$$

To determine C plugin the center $x=0$

$f(0) = \ln(2-0) = 0 + C \Rightarrow C = \ln 2$

$$\boxed{f(x) = \ln(2-x) = \ln 2 - \sum_{n=0}^{\infty}\frac{x^{n+1}}{2^{n+1}(n+1)}}, \quad \boxed{R=2}$$

18. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 3^n}$ converges to s. Use the Alternating Series Theorem to estimate $|s - s_6|$.

$$s = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 3^n}$$

$$|s - s_6| = |R_6|$$

We know $\sum_{n=1}^{\infty} (-1)^n b_n$

$$|R_n| \leq b_{n+1}$$

In our case $b_n = \frac{1}{n^2 3^n}$

$$|R_6| \leq b_{6+1} = b_7 = \boxed{\frac{1}{7^2 \cdot 3^7}}$$