# Fall 2012 Math 152

Week in Review 2

courtesy: *Oksana Shatalov*

(covering Section 7.2 )

## 7.2: VOLUME

### Key Points:

*Cross sections are perpendicular to the axis of rotating. $A(x)$ is crossectional area.*

- $V = \int_a^b A(x)\,\mathrm{d}x$ where $A(x)$ is the area of a moving cross-section obtained by
  . slicing through $x$ **perpendicular** to the $x$-axis.
- $V = \int_c^d A(y)\,\mathrm{d}x$ where $A(y)$ is the area of a moving cross-section obtained by
  . slicing through $y$ **perpendicular** to the $y$-axis.

### Examples

1. Find the volume of the solid whose base is the region in the first quadrant bounded by parabola $x = y^2$ and line $x = 9$ and whose cross-sections are semicircles with the diameter perpendicular to the $x$-axis.
2. Find the volume of the solid whose base is a unit disk and whose cross-sections are squares with the base perpendicular to the $y$-axis.
3. Find the volume of the solid whose base is the region bounded by $x = 4 - y^2$ and line $x = y^2 - 4$ and whose cross-sections are equilateral triangles with the base perpendicular to the $y$-axis.
4. Determine the volume of the solid whose base is an elliptical region with boundary curve $\frac{x^2}{4} + y^2 = 1$ and whose cross-sections perpendicular to the $x$ axis are isosceles right triangles with hypotenuse in the base.

### Key Points: *DISK METHOD*

*Cross sections perpendicular to the axis of rotating are* **disks** *and* $A(x) = \pi(radius)^2$.

- A solid is formed by rotating a curve about a horizontal axis ($y = k$): $A(x) = \pi\,[r(x)]^2$.
- A solid is formed by rotating a curve about a vertical axis ($x = k$): $A(y) = \pi\,[r(y)]^2$ .

## Examples

5. Determine the volume of the solid obtained by rotating the region bounded by $y = \sqrt{x}$, $y = 4$ and the $y$-axis about the $y$-axis.

6. Determine the volume of the solid obtained by rotating the region

$$D = \left\{(x, y) \mid \quad -3 \le x \le 3, \quad 0 \le y \le 10 - x^2\right\}$$

about the $x$-axis.

7. Determine the volume of the solid obtained by rotating the region bounded by $x = 5 - y^2$, $x = 1$ about the line $x = 1$.

8. Sketch and find the volume of the solid obtained by rotating the region under the graph of $f(x) = \begin{cases} -x & \text{if} \quad -2 \le x < -1 \\ 1 & \text{if} \quad -1 \le x \le 0 \\ e^x & \text{if} \quad 0 < x \le 3 \end{cases}$ about the $x$-axis.

9. Each integral represents the volume of a solid. Describe the solid.

   (a) $\pi \displaystyle\int_0^{\pi/6} \cos^2 x \, dx$

   (b) $\pi \displaystyle\int_3^{10} x^4 \, dx$

## Key Points: *WASHER METHOD*

*Cross sections perpendicular to the axis of rotating are* **washers** *(rings) and*

$$A(x) = \pi \left[ \begin{pmatrix} \textit{outer} \\ \textit{radius} \end{pmatrix}^2 - \begin{pmatrix} \textit{inner} \\ \textit{radius} \end{pmatrix}^2 \right]$$

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- If a solid is formed by rotating about a horizontal axis ($y = k$) then $A(x) = \pi\left[R(x)^2 - r(x)^2\right]$
- If a solid is formed by rotating about a vertical axis ($x = k$) then $A(y) = \pi\left[R(y)^2 - r(y)^2\right]$

## Examples

10. Determine the volume of the solid obtained by rotating the region bounded by $y = 3x^2$ and $y = \frac{x^3}{3}$ about the $x$-axis.

11. Determine the volume of the solid obtained by rotating the region bounded by $y = 3x^2$ and $y = \frac{x^3}{3}$ about the $y$-axis.

12. Determine the volume of the solid obtained by rotating triangle with vertices $(0, 0)$, $(1, 0)$ and $(1, 4)$ about the line $y = -5$.

13. Determine the volume of the solid obtained by rotating the region bounded by $x = y^2 - 6y + 12$ and $x = 7$ about the $y$-axis.

14. Set up the integral representing the volume of the solid obtained by rotating the region bounded by $y = e^{-x}$, $y = \sqrt{x} + 1$ and $x = 1$ about the line $y = -3$.

15. Each integral represents the volume of a solid. Describe the solid.

    (a) $\pi \int_0^{\sqrt{5}} (25 - y^4)\, \mathrm{d}y$

    (b) $\pi \int_{\pi/4}^{\pi/2} \left[(2 + \sin x)^2 - (2 + \cos x)^2\right] \mathrm{d}x$