

## Fall 2012 Math 152

### Week in Review 3

courtesy: *Oksana Shatalov*

(covering Sections 7.3, 7.4& 7.5 )

## 7.3: Volumes by Cylindrical Shells

### Key Points

- Area of cross sections:  $A = 2\pi(\text{radius})(\text{height})$ .
  - For rotation about a *vertical* axis we use  $V = \int_a^b A(x) dx = 2\pi \int_a^b r(x)h(x) dx$ .  
Note  $h(x)=(\text{Top}-\text{Bottom})$ .
  - For rotation about a *horizontal* axis we use  $V = \int_c^d A(y) dy = 2\pi \int_c^d r(y)h(y) dy$ .  
Note  $h(y)=(\text{Right}-\text{Left})$
- In many problems both washer method and shell method can be used. In some problems both take about the same amount of work, but sometimes one is definitely easier than the other.

### Examples

1. Find the volume generated by rotating the region bounded by the given curve(s).
  - (a)  $y = \frac{1}{x^4}$ ,  $x = \frac{1}{2}$ ,  $x = 1$ ,  $y = 0$ . Rotate about the  $y$ -axis.
  - (b)  $y = x^3$ ,  $x = 0$ ,  $y = 27$ . Rotate about the  $x$ -axis.
  - (c)  $y = \sqrt{x}$ ,  $y = 4x$ . Rotate about the  $y$ -axis.
2. Find the volume generated by rotating the region bounded by the given curve(s) about the specified lines. Do not evaluate either integral. Just set up.
  - (a)  $y = \sin x$ ,  $x = \pi/4$ ,  $x = \pi/2$ ,  $y = 0$ .
    - i. Rotate about the the line  $x = -\pi/4$ .
    - ii. Rotate about the the line  $x = \pi$ .
  - (b)  $y = \ln x$ ,  $x = e$ ,  $y = 0$ .
    - i. Rotate about the line  $y = 2$ .
    - ii. Rotate about the line  $y = -2$ .
  - (c)  $y = \sin x$  on the interval  $[0, \pi]$ ,  $y = \frac{1}{2}$ . Rotate about the line  $x = -\frac{\pi}{3}$ .

## 7.4: Work

### Key Points

- Work  $W$  done in moving an object under
  - a constant force  $F$  a distance  $d$  is  $W = Fd$ .
  - under a variable force  $F(x)$  ( $x$  is displacement) from  $x = a$  to  $x = b$  is  $W = \int_a^b F(x) dx$
- Hooke's law: the force required to stretch a spring  $x$  units beyond its natural length is  $F = kx$ .

- "Water Pumping" problems require partition along vertical axis (called  $y$  here) and work is given by

$$W = \rho g \int_a^b (\text{Area})(\text{distance}) dy$$

where

- $y = 0$  can be top, bottom, or center (circular ends);
  - $\rho g$  is weight density of liquid;
  - (Area) is cross sectional area of horizontal slice;
  - (distance) is the distance the slice travels to reach the top.
- "Rope/cable pulling" problems require partition along vertical axis (called  $y$  here). If  $y = 0$  is at the top of rope/cable then a work required to pull  $s$  feet (or  $s$  meter) of a rope/cable that weighs  $\omega$  lb/ft (or  $\omega$  N/m) is given by

$$W = \int_0^s \omega y dy$$

### Examples

3. A force of  $F(x) = x^4 - \sin(4\pi x) + 12$  (where  $x$  is in meters) acts on an object. What is the work required to move the object from  $x=3$  to  $x = 5$  ?
4. If the force required to stretch a spring 3ft beyond its natural length is 12lb, how much work is needed to stretch it 15 inches beyond its natural length?
5. A spring has a natural length of 2m. If a 10J work is required to keep it stretched to a length 2.5m, how much work is done in stretching the spring from 3m to 8m?
6. A tank of water is 20 feet long and has a vertical cross section in the shape of an equilateral triangle with sides 2 feet long (point of the triangle points directly down). The tank is filled with water to a depth of 18 inches. Determine the amount of work needed to pump all of the water to the top of the tank. Assume that the density of water is  $62.5\text{lb/ft}^3$ .

7. A spherical tank with radius 8m is  $1/2$  full of water. The water pumped from a spout at the top of the tank that is 75cm high. Find the work needed to pump out the water (through the spout) until the water level is 4m from the bottom. (The density of water is  $1000\text{kg}/\text{m}^3$ .)
8. A heavy rope 40ft long, weighs  $0.4\text{lb}/\text{ft}$  and hangs over the edge of a tall building. How much work is done in pulling the rope to the top of the building?
9. A uniform cable hanging over the edge of a tall building is 6m long and weight  $20\text{kg}$ . If a  $25\text{kg}$  weight is attached to the end of the cable, how much work is required to pull 2m of the cable to the top? Use  $g \approx 10\text{m}/\text{s}^2$  for the gravitational constant.

## 7.5: Average Value of a Function

### Key Points

- The **average value of a function**  $y = f(x)$  over the interval  $[a, b]$ :

$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) dx.$$

- MEAN VALUE THEOREM FOR INTEGRALS: *If  $f$  is continuous on  $[a, b]$ , then there exists a number  $c$  on  $[a, b]$  s.t.*

$$\int_a^b f(x) dx = f(c)(b-a).$$

### Examples

10. Determine the average value of  $f(x) = \frac{\sqrt{3}-1}{1+x^2}$  over the interval  $[1, \sqrt{3}]$ .
11. Find the value(s)  $b$  such that the average value of  $f(x) = 3x^2 - 2x - 3$  on the interval  $[a, 0]$  is equal to 1.