Fall 2012 Math 152

Week in Review 6

courtesy: *Oksana Shatalov*

(covering Section 9.3, 9.4 &10.1)

9.3: Arc Length

Key Points

- Arc length of curve C: $\int \mathrm{d}s$
- $\int \mathrm{d}s = \int_\alpha^\beta \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\mathrm{d}t$ when C is given by $x = x(t), y = y(t), \alpha \le t \le \beta$;
- $\int \mathrm{d}s = \int_a^b \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\mathrm{d}x$ when C is given by $y = y(x)$, $a \le x \le b$;
- $\int \mathrm{d}s = \int_c^d \sqrt{1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\mathrm{d}y$ when C is given by $x = x(y)$, $c \le x \le d$.

Examples

1. Find the length of the curve $x = \cos^3 t, y = \sin^3 t$, $0 \le t \le \pi/2$.
2. Find the length of the curve $x = \frac{1}{4}\ln y - \frac{1}{2}y^2$ from $y = 1$ to $y = e$.
3. A wire hanging between two poles (at $x = -10$ and $x = 10$) takes the shape of a catenary with equation
$$y = 2(e^{x/4} + e^{-x/4}).$$
Find the length of the wire.

9.4: Area of a Surface of Revolution

Key Points

- $SA = 2\pi \int (radius)\mathrm{d}s$
- about the x-axis: $radius = y$
- about the y-axis: $radius = x$

Examples

4. The curve $y = x^2$, $0 \le x \le 1$, is rotated about the y-axis. Find the area of the resulting surface.

5. The curve $x = 1 - \cos(2t)$, $y = 2t + \sin(2t)$, $0 \le t \le \pi/4$ is rotated about the x-axis. Find the area of the resulting surface.

6. Set up (but don't evaluate) the integral that gives the surface area obtaine by rotating the curve
$$x = \sin(\pi y^2/8), \quad 1 \le y \le 2,$$
   (a) about the x-axis
   (b) about the y-axis

7. The curve $x = \sin(at), y = \cos(at), 0 \le t \le \frac{\pi}{2a}$ is rotated about the x-axis (here a is an arbitrary positive constant). Find the area of the resulting surface.

10.1: Sequences

Key Points

- If $\lim_{n\to\infty} a_n$ exists and finite then we say that the sequence $\{a_n\}$ **converges**. Otherwise, we say the sequence **diverges**. (Recall all techniques for finding limits at infinity.)
- The *Squeeze* Theorem for Sequences: If $a_n \le b_n \le c_n$ for all n and the sequences $\{a_n\}$ and $\{c_n\}$ have a common limit L as $n \to \infty$, then $\lim_{n\to\infty} b_n = L$.
- If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.
- $\{a_n\}$ *increasing:* show that $a_{n+1} - a_n > 0$, or $f'(x) > 0$ (where $f(n) = a_n$); or $\dfrac{a_n + 1}{a_n} > 1$ (provided $a_n > 0$ for all n.) Note: reverse signs for $\{a_n\}$ *decreasing*.

Examples

8. Define the n-th term of the sequence $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots\right\}$ and find its limit.

9. Determine if the given sequence converges or diverges. If it converges, find the limit.
   (a) $a_n = \dfrac{3n^5 - 12n^3 + 2012}{2012 - 12n^4 - 4n^4 - 9n^5}$
   (b) $b_n = \dfrac{3n^5 - 12n^3 + 2012}{2012 - 12n^4 - 4n^4 - 9n^5 + 11n^6}$

(c) $c_n = \dfrac{12n^7 + 2012}{2012 - 12n^4 - 4n^5 - 9n^6}$

10. Determine if the sequence with the given general term ($n \geq 1$) converges or diverges. If it converges, find the limit.

    (a) $a_n = \ln(n^2 + 3) - \ln(7n^2 - 5)$

    (b) $z_n = \dfrac{1}{n^4} \sin(\dfrac{1}{n^5})$

    (c) $y_n = \dfrac{(-1)^n}{n^3}$

    (d) $x_n = \dfrac{(-1)^n n}{3n + 33}$

    (e) $a_n = \dfrac{(\arctan n)^7}{n^5}$

11. Assuming that the sequence defined recursively by $a_n = 1$, $a_{n+1} = \frac{1}{2}\left(a_n + \frac{9}{a_n}\right)$ is convergent, find its limit.

12. Determine whether the given seqence is increasing or decreasing.

    (a) $\{\arctan(n)\}_{n=1}^{\infty}$

    (b) $\{n - 2^n\}_{n=1}^{\infty}$

    (c) $\left\{\dfrac{10^n}{n!}\right\}_{n=1}^{\infty}$