Math 171

Sample problems for Test 1.

1. A woman walks due west on a ship at 4 mph. The ship is moving N30^0W at 20 mph. (This means that the direction from which the ship moves is 30^0 west of the northerly direction.) Find the speed of the woman relative to the water.

2. Find $f'(x)$ for $f(x) = \sqrt{5-x}$ using the definition of derivative.

3. Prove Properties of the Dot Product.(see Theorem 3 in notes, section 1.2)

4. Given vectors $\vec{a} =< 4, 6 >$ and $\vec{b} =< -3, 2 >$.

   (a) Find the unit vector in the direction of $\vec{b}$

   (b) Find the angle between $\vec{a}$ and $\vec{b}$

   (c) Find the scalar and the vector projections of $\vec{a}$ onto $\vec{b}$.

5. Find the equation of the line that passes through the point (1,3) and is perpendicular to the vector $\vec{n} = -4i + \jmath$.

6. Find the distance from the point (-5,2) to the line $x - 2y = 4$.

7. Find the work done by a force of 20 lb acting in the direction N50^0W in moving an object 4 ft due west.

8. Find the Cartesian equation of the curve given by $x = \cos t$, $y = \cos 2t$, $0 \leq t < 2\pi$.

9. Find the vector and parametric equations of a line that passes through the points (1,2) and (-3,4).

10. A particle is moving in the xy-plane and its position (x, y) at time t is given by $x = 3t + 1$, $y = t^2 - t$.

    (a) Find the position of the particle at time $t = 3$.

    (b) At what time is the particle at the point (16,20)?

11. Prove using the ε, δ definition of limit that $\lim_{x\to 2}(3x - 2) = 4$.

12. Prove that

    (a) $(\cos x)' = -\sin x$

    (b) $(\sin x)' = \cos x$

    (c) $(\tan x)' = \sec^2 x$

    (d) $(\sec x)' = \sec x \tan x$

13. Find the limit if it exists:

    (a) $\lim_{t\to 1} \dfrac{t^3 - 1}{t^2 - 1}$

    (b) $\lim_{x\to 5} \dfrac{x^2 - 5x + 10}{x^2 - 25}$

    (c) $\lim_{x\to 7} \dfrac{2 - \sqrt{x-3}}{x^2 - 49}$

    (d) $\lim_{t\to 1} \left\langle \dfrac{t^2 - 2t + 1}{t - 1}, \dfrac{\sqrt{t} - 1}{t^2 - 1} \right\rangle$

    (e) $\lim_{x\to -3} |x + 3|$

    (f) $\lim_{y\to\infty} \dfrac{7y^3 + 4y}{2y^3 - y^2 + 3}$

    (g) $\lim_{x\to\infty} (\sqrt{x^2 + 3x + 1} - x)$

(h) $\displaystyle\lim_{x\to 0} \frac{\sin(5x)}{\tan(2x)}$

14. Use the Squeeze Theorem to prove that $\lim_{x\to 0} \sqrt{x}\cos^4 x = 0$.

15. Find the horizontal and vertical asymptotes of the curve $y = \dfrac{x^2+4}{3x^2-3}$.

16. Use the definition of continuity to show that the function $f(x) = \dfrac{x+1}{2x^2-1}$ is continuous at $a = 4$.

17. Find the values of c and d that make the function

$$f(x) = \begin{cases} 2x, & \text{if } x < 1 \\ cx^2 + d, & \text{if } 1 \le x \le 2 \\ 4x, & \text{if } x > 2 \end{cases}$$

continuous on $(-\infty, \infty)$.

18. For each of the functions below, find all points of discontinuity, and classify them as removable discontinuities, jump discontinuities, or infinity discontinuities:

(a) $f(x) = \dfrac{x^2-2x-8}{x+2}$

(b) $g(x) = \dfrac{5x-3}{x^2-4}$

(c) $h(x) = \begin{cases} 1-x, & \text{if } x \le 2 \\ x^2-2x, & \text{if } x > 2 \end{cases}$

19. Use the Intermediate Value Theorem to show that there is a root of the equation $x^3 - 3x + 1 = 0$ in the interval (1,2).

20. Find the derivative f' of each function:

(a) $f(x) = (x^9 + x^7 - 5x^2 + 2013)(x^2 - 4x + 6)$

(b) $f(x) = \dfrac{x^7+9x-13}{\sqrt{x}-3}$

(c) $f(x) = x + x\sqrt[5]{x^2} + x\sqrt{x} + \dfrac{1}{x\sqrt{x}} + 15$

(d) $f(x) = \sqrt{\tan x} + \cos^2(\tan^2 x + 6)$

21. Find the equation of the tangent line to the curve $y = x\sqrt{5-x}$ at the point $(1, 2)$.

22. Find parametric equations for the line tangent to the curve $r =< t^2+2t, t^3-t >$ at the point corresponding to $t = 1$.