Final Exam Practice

In addition to working this problem set, it is advised that you work the first two exams, quizzes, as well as Lecture Notes.

1. Given $\mathbf{a} = \langle -2, 3\rangle$, $\mathbf{b} = \langle 6, 1\rangle$, $\mathbf{c} = 2\mathbf{i}$. Find:
   (a) $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c})$
   (b) a unit vector having the same direction as $\mathbf{a}$
   (c) Find the angle between $\mathbf{a}$ and $\mathbf{b}$
   (d) a unit vector that is orthogonal to $\mathbf{a} + \mathbf{b}$
   (e) scalars α and β such that $\mathbf{c} = \alpha\mathbf{a} + \beta\mathbf{b}$
2. Find a vector equation of the line containing the points $(-1, 1)$ and $(2, 5)$.
3. Find a unit vector perpendicular to the line described by the parametric equations $x = -4t + 1$, $y = 6t + 5$.
4. Find the work done by a force of $30N$ acting in the direction $N30^oW$ (i.e. 30^o west of the northerly direction) in moving an object $6m$ due west.
5. Determine whether the vectors $\langle 1, 2\rangle$ and $\langle -2, 3\rangle$ are orthogonal, parallel, or neither.
6. What is the limit:
   (a) $\displaystyle\lim_{\theta\to\pi/3} \frac{\cos\theta - \frac{1}{2}}{\theta - \pi/3}$
   (b) $\displaystyle\lim_{h\to 0} \frac{(2+h)^6 - 64}{h}$
   (c) $\displaystyle\lim_{h\to 0} \frac{\sin(\pi/4 + h) - \sin(\pi/4)}{h}$
7. Compute the following limits:
   (a) $\displaystyle\lim_{x\to -6^+} \frac{x}{x+6}$
   (b) $\displaystyle\lim_{x\to 16} \frac{4 - \sqrt{x}}{x - 16}$
   (c) $\displaystyle\lim_{x\to 8^-} \frac{|x-8|}{x-8}$
   (d) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^2 - 9}}{2x - 6}$
   (e) $\displaystyle\lim_{x\to\infty} (\sqrt{x^2 + x + 1} - \sqrt{x^2 - x})$
   (f) $\displaystyle\lim_{x\to\infty} \frac{1 + 2x - x^2}{1 - x + 2x^2}$

(g) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{\tan 3x}$

(h) $\lim\limits_{x\to 0} \dfrac{\cos x - 1}{\sin 5x}$

(i) $\lim\limits_{x\to 0} x \sin \dfrac{1}{x}$

(j) $\lim\limits_{x\to 0} \dfrac{1+\sin x - \cos x}{1 - \sin x - \cos x}$

(k) $\lim\limits_{x\to \pi/2} \left(\dfrac{\pi}{2} - x\right) \tan x$

(l) $\lim\limits_{x\to \infty} \left(1 + \dfrac{1}{x} + \dfrac{1}{x^2}\right)^x$

8. Discuss the continuity of

$$f(x) = \begin{cases} 2x - x^2 & \text{if} \quad 0 \le x \le 2 \\ 2 - x & \text{if} \quad 2 < x \le 3 \\ x - 4 & \text{if} \quad 3 < x < 4 \\ \pi & \text{if} \quad x \ge 4 \end{cases}$$

9. Find all horizontal and vertical asymptotes of the curve $y = \dfrac{x}{\sqrt[4]{x^4+1}}$

10. Given the curve $y = \dfrac{2}{1-3x}$. Find:

   (a) the slope of the tangent line to this curve at the point $(2, 1)$;

   (b) the equation of this tangent line.

11. Find $f^{(5)}(0)$ for

   (a) $f(x) = 2^x$.

   (b) $f(x) = e^{2x}$.

12. What is the domain of $f(x) = \log_5(5 - e^x)$?

13. Calculate y' for

   (a) $x^2y^3 + 3y^2 = x - 4y$

   (b) $\cos(x + 2y) = 4x^2 - y^3$

14. Compute the derivative:

   (a) $y = \dfrac{(x+5)^4}{x^4 + 5^4}$

   (b) $y = \dfrac{1}{\sin(x - \sin x)}$

(c) $y = \tan^5(\sqrt{1-x^2})$

(d) $y = \ln(\cos x)$

(e) $y = \arccos(\sqrt{t}) + \arctan(5t)$

15. Suppose that $h(x) = f(x)g(x)$ and $F(x) = f(g(x))$, where $f(2) = 3, g(2) = 5, g'(2) = 4, f'(2) = -2, f'(5) = 11$. Find $h'(2)$ and $F'(2)$.

16. If $H(x) = f(x^2 + 4x)$ and $f'(12) = 7$ find $H'(2)$.

17. Find the equation of the tangent to the curve $y = \ln(e^x + e^{2x})$ at the point $(0, \ln 2)$.

18. At what point on the curve $y = [\ln(x+4)]^2$ is the tangent line horizontal?

19. Find the linear approximation for $f(x) = \sqrt{25 - x^2}$ near 3.

20. The volume of a cube is increasing at a rate of 10cm^3/min. How fast is the surface area increasing when the length of the edge is 80cm.

21. A paper cup has the shape of cone with height 10cm and radius 3cm at the top. If water is poured into the cup at a rate of 2cm^3/s, how fast is the water level rising when the water is 5cm deep?

22. A balloon is rising at a constant speed of 5ft/s. A boy is cycling along a straight road at a speed of 15ft/s. When he passed under the balloon it is 45ft above him. How fast is the distance between the boy and the balloon increasing 3s later?

23. Solve each equation for x:

(a) $e^{e^x} = 2$

(b) $\ln(x+1) - \ln x = 1$

(c) $3^t = 9^{2t-1}$

24. Given $\mathbf{r}(t) = \ln t\mathbf{i} + te^{2t}\mathbf{j}$. Find parametric equations for the tangent line to the curve at the point $(0, e^2)$.

25. If $f(x) = 3x^4 - 4x^3 - 12x^2 + 2$ find the intervals where $f(x)$ is increasing or decreasing and locate all local extrema.

26. Where is $f(x) = x \ln x$ concave up?

27. Find the absolute extreme values for $f(x) = x^3 - 12x + 5$ over the interval $[-5, 1]$.

28. Find the most general antiderivative of $\dfrac{1+4x}{\sqrt{x}}$.

29. Find $f(x)$ if $f'(x) = 1 + 2\sin x - \cos x$, $f(0) = 3$.

30. Compute
    (a) $\sin(2\arcsin\frac{3}{5})$
    (b) $\arcsin(\sin\frac{5\pi}{4})$

31. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 8 - x^2$.

32. Evaluate:
    (a) $\displaystyle\int_0^{\pi/2} \frac{\mathrm{d}}{\mathrm{d}x}\left(\sin\frac{x}{2}\cos\frac{x}{2}\right)\mathrm{d}x$
    (b) $\displaystyle\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_x^{\pi/2}\sin\frac{t}{2}\cos\frac{t}{2}\,\mathrm{d}t\right)$

33. Evaluate the integral if it exists:
    (a) $\displaystyle\int_1^8 \sqrt[3]{x}(x-1)\,\mathrm{d}x$
    (b) $\displaystyle\int_0^b (x^3+4x-1)\,\mathrm{d}x$
    (c) $\displaystyle\int_1^4 \frac{x^2-x+1}{\sqrt{x}}\,\mathrm{d}x$
    (d) $\displaystyle\int_{-1}^2 (x-2|x|)\,\mathrm{d}x$

34. Find the area under the curve $y = 8e^x$ from $\ln(3)$ to $\ln(6)$.

From textbook:

1. page 146 problems 1-3
2. page 234 problems 1-6, 9-12
3. page 297 problems 2-9, 11, 12
4. page 356 problems 1,2,5-7, 9-12
5. page 416 problems 1-16, 13