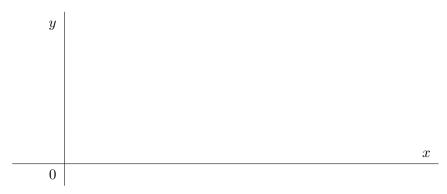
## Section 2.7: Tangents, velocities, and other rates of change.

Consider a curve with equation y = f(x) and points P(a, f(a)) and Q(x, f(x)) on it. The slope of the secant line PQ (also known as **average rate** or **average velocity**) is

$$m_{PQ} = \frac{f(x) - f(a)}{x - a}$$



DEFINITION 1. The tangent line to the curve y = f(x) at the point P(a, f(a)) is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \tag{1}$$

provided that this limit exists.

The slope (1) of tangent line also known as **instantaneous rate of change** or **instantaneous velocity**.

EXAMPLE 2. (a) Find the slope of the tangent line to the graph of  $f(x) = x^2 - 3x - 4$  at (5,6).

(b) Find the equation of the tangent line to the graph of f(x) at x = 5. (Recall that point-slope form for a line through the point  $(x_1, y_1)$  with slope m is:  $y - y_1 = m(x - x_1)$ .)

DEFINITION 3. If  $\mathbf{r}(t)$  is a vector function then the tangent vector  $\mathbf{v}$  at t = a is found by

$$\mathbf{v} = \lim_{t \to a} \frac{1}{t - a} [\mathbf{r}(t) - \mathbf{r}(a)].$$

EXAMPLE 4. Given curve  $\mathbf{r}(t) = \langle 2t, 10t - t^2 \rangle$ .

(a) Find a vector tangent to the curve at the point (4,16).

(b) Find parametric equations of the tangent line to  $\mathbf{r}(t)$  at t=2.

(c) Find a cartesian equation of this tangent line.

**Velocities.** Denote by f(t) the position of an object at time t. The **Average Velocity** of the object from t = a to t = b is

$$\frac{f(b) - f(a)}{b - a}.$$

The **Instantaneous Velocity** of the object at time t = a is

$$v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

(Compare this formula with (1) by substitution h = x - a.)

EXAMPLE 5. The position (in meters) of an object moving in a straight path is given by

$$s(t) = t^2 - 2t + 8,$$

where t is measured in seconds.

(a) Find the average velocity over the time interval [4,5].

(b) Find the instantaneous velocity at time t = 4.

## Other Rates of Change:

The **Average Rate** of change of function f(x) from x = a to x = b is

$$\frac{f(b) - f(a)}{b - a}.$$

The Instantaneous Rate of Change of f(x) at x = a is

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$