Section 3.10: Related rates

In this section, we have two or more quantities that are changing with respect to time t. We will apply the following strategy:

- 1. Read the problem carefully and draw a diagram if possible.
- 2. Express the given information and the required rates in terms of derivatives and state your "find" and "when".
- 3. Find a formula (equation) that relates the quantities in the problem. (If necessary, use Geometry¹ of the situation to eliminate one of the variables by substitution.) Don't substitute the given numerical information at this step!!!
- 4. Use the Chain Rule to differentiate both sides of the equation with respect to t.
- 5. Substitute the given numerical information in the resulting equation and solve for the desired rate of change.

EXAMPLE 1. A spherical balloon is inflated with gas at a rate of 25ft³/min. How fast is the radius changing when the radius is 2ft?

• Circle:
$$A = \pi r^2$$
; $C = 2\pi r$

• Sector of Circle:
$$A = \frac{1}{2}r^2\theta; \ s = r\theta$$

$$\bullet \ \, {\rm Sphere} \colon \, V = \frac{4}{3} \pi r^3; \, A = 4 \pi r^2$$

$$\bullet \ \ {\rm Cone:} \ \ V = \frac{1}{3}\pi r^2 h$$

¹Useful formulas:

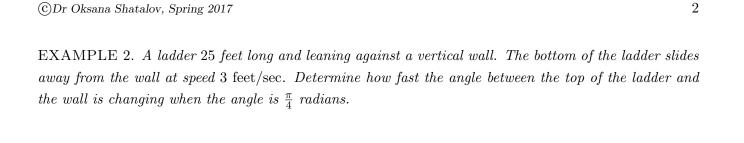
[•] Triangle: $A = \frac{1}{2}bh$

 [—] Equilateral Triangle: $h=\frac{\sqrt{3}s}{2};\,A=\frac{\sqrt{3}s^2}{4}$ — Right Triangle: Pythagorean Theorem $c^2=a^2+b^2$

[•] Trapezoid: $A = \frac{h}{2}(b_1 + b_2)$

Parallelogram: A = bh

[•] Cylinder: $V = \pi r^2 h$



EXAMPLE 3. A water tank has the shape of an inverted right circular cone with height 16m and base radius 4m. Water is pouring into the tank at $3\text{m}^3/\text{min}$. How fast is the water level rising when the water in the tank is 5 meters deep?

EXAMPLE 4. Two people are separated by 350 meters. Person A starts walking north at a rate of 0.6 m/sec and 7 minutes later Person B starts walking south at 0.5 m/sec. At what rate is the distance separating the two people changing 25 minutes after Person A starts walking?

EXAMPLE 5. A plane flying with a constant speed of 360km/hour passes over a radar station at an altitude of 2km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing 1 minute later?