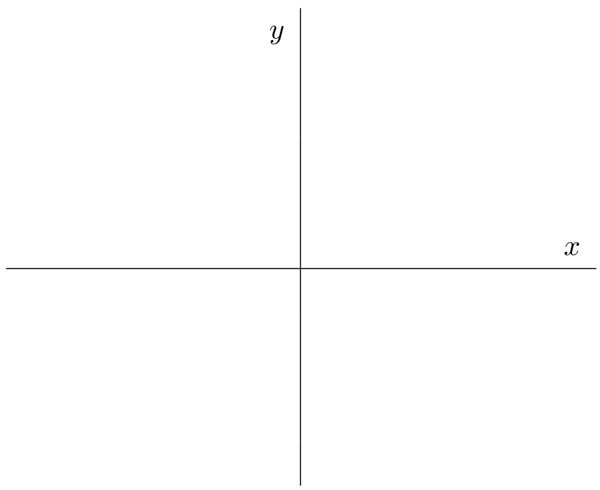
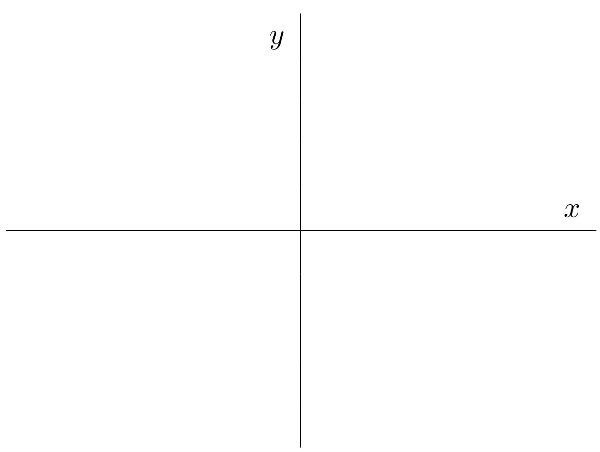
3.7: Derivatives of the vector functions

EXAMPLE 1. *Sketch the curve* $\mathbf{r}(\mathbf{t})$ *and indicate with arrow the direction in which* t *increases if*

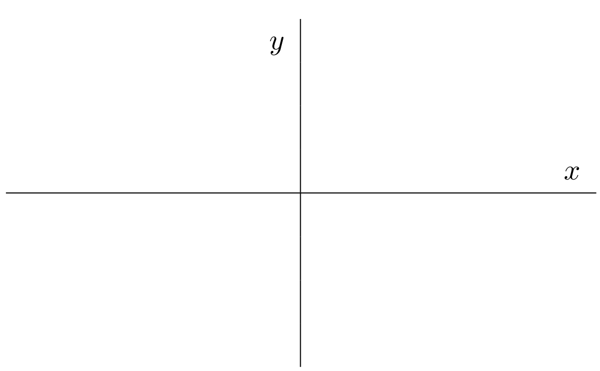
(a) $\mathbf{r}(\mathbf{t}) = \left\langle t^2, t \right\rangle$



(b) $\mathbf{r}(\mathbf{t}) = \langle 2\sin t, \cos t \rangle$



(c) $\mathbf{r}(\mathbf{t}) = \langle 1 + 2\sin t, \cos t \rangle$



DEFINITION 2. *If* $(r)(t) = \langle x(t), y(t) \rangle$ *is a vector function, then*

$$\mathbf{r}'(t) = \left\langle x'(t), y'(t) \right\rangle$$

if both $x'(t), y'(t)$ *exist.*

EXAMPLE 3. *If* $\mathbf{r}(\mathbf{t}) = \left\langle t^2, \sqrt{t-5} \right\rangle$ *find the domain of* $\mathbf{r}(t)$ *and* $\mathbf{r}'(t)$*.*

DEFINITION 4. *If* $(r)(t) = \langle x(t), y(t) \rangle$ *is a vector function representing the position of a particle at time* t*, then*

- **instantaneous velocity** *at time* t *is* $\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle$
- **instantaneous speed** *at time* t *is* $|\mathbf{r}'(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$

EXAMPLE 5. *The vector function* $\mathbf{r}(t) = \left\langle t, \sqrt{t^2+9} \right\rangle$ *represents the position of a particle at time* t*. Find the velocity and speed of the particle at time* $t = 4$*.*

DEFINITION 6. *The* **angle between two intersecting curves** *(curvilinear angle) is defined to be the angle between the tangent lines at the point of intersection.*

EXAMPLE 7. *Given two curves traced by*

$$\mathbf{r}(t) = \left\langle 1+t, 3+t^2 \right\rangle, \qquad \mathbf{R}(s) = \left\langle 2-s, s^2 \right\rangle.$$

(a) *At what point do the curves intersect?*

(b) *Find the angle between the curves.*