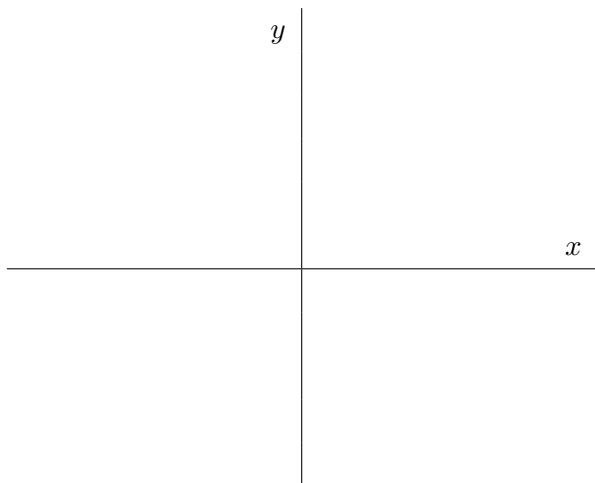


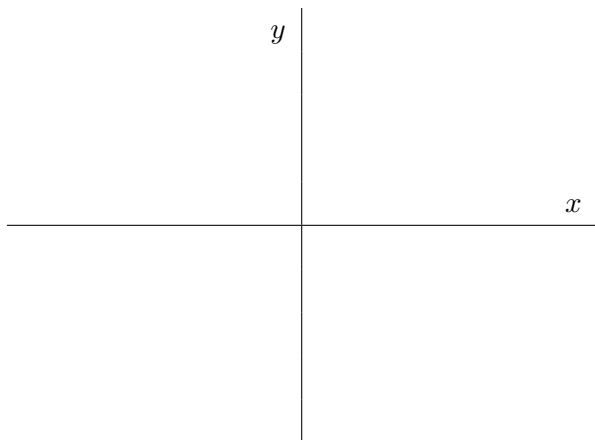
3.7: Derivatives of the vector functions

EXAMPLE 1. Sketch the curve $\mathbf{r}(t)$ and indicate with arrow the direction in which t increases if

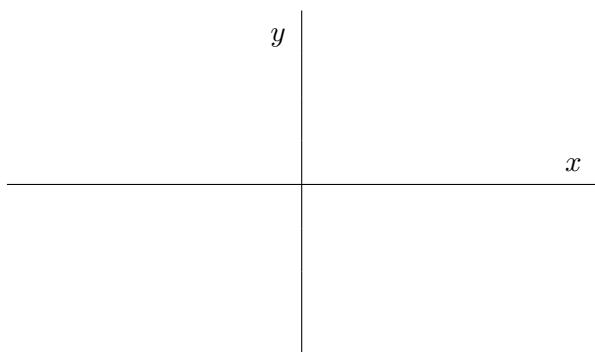
(a) $\mathbf{r}(t) = \langle t^2, t \rangle$



(b) $\mathbf{r}(t) = \langle 2 \sin t, \cos t \rangle$



(c) $\mathbf{r}(t) = \langle 1 + 2 \sin t, \cos t \rangle$



DEFINITION 2. If $\mathbf{r}(t) = \langle x(t), y(t) \rangle$ is a vector function, then

$$\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle$$

if both $x'(t), y'(t)$ exist.

EXAMPLE 3. If $\mathbf{r}(t) = \langle t^2, \sqrt{t-5} \rangle$ find the domain of $\mathbf{r}(t)$ and $\mathbf{r}'(t)$.

DEFINITION 4. If $(r)(t) = \langle x(t), y(t) \rangle$ is a vector function representing the position of a particle at time t , then

- **instantaneous velocity** at time t is $\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle$
- **instantaneous speed** at time t is $|\mathbf{r}'(t)| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$

EXAMPLE 5. The vector function $\mathbf{r}(t) = \langle t, \sqrt{t^2 + 9} \rangle$ represents the position of a particle at time t . Find the velocity and speed of the particle at time $t = 4$.

DEFINITION 6. The **angle between two intersecting curves** (*curvilinear angle*) is defined to be the angle between the tangent lines at the point of intersection.

EXAMPLE 7. Given two curves traced by

$$\mathbf{r}(t) = \langle 1 + t, 3 + t^2 \rangle, \quad \mathbf{R}(s) = \langle 2 - s, s^2 \rangle.$$

(a) At what point do the curves intersect?

(b) Find the angle between the curves.