3.9: Slopes and tangents of parametric curves

Consider a curve C given by the parametric equations

$$x = x(t), \quad y = y(t),$$

or in vector form:

$$\mathbf{r}(t) = \langle x(t), y(t) \rangle .$$

If both $x(t)$ and $y(t)$ are differentiable, then

$$\mathbf{r}'(t) = \langle x'(t), y'(t) \rangle$$

is a vector that is tangent to C. Its slope is:

$$\text{slope} =$$

Another way to see this is by using the Chain Rule. We have $y = y(x(t))$ and then

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

which implies

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}.$$

EXAMPLE 1. *Find the equation of tangent line to the curve*

$$x(t) = \sin t, \quad y(t) = \tan t$$

at the point corresponding to $t = \dfrac{\pi}{4}$.

EXAMPLE 2. *Find the equation of tangent line to the curve*

$$x(t) = t + 1, \quad y(t) = t^2 + 4$$

at the $(2, 5)$.

EXAMPLE 3. *Find the points on the curve*

$$x = t + t^2, \quad y = t^2 - t$$

where the tangent lines are horizontal and there they are vertical.

REMARK 4. It may happen that $x'(t) = y'(t) = 0$ for some value of t.

Illustration 1. $x(t) = t^3,\ y(t) = t^3$

Illustration 2. $x(t) = t^3,\ y(t) = t^2$

EXAMPLE 5. *Show that the curve*

$$x = \cos t, \quad y = \cos t \sin t$$

has two tangents at $(0, 0)$ *and find their equations.*