4.3:Logarithmic Functions

DEFINITION 1. The exponential function $f(x) = a^x$ with $a \neq 1$ is a one-to-one function. The inverse of this function, called the logarithmic function with base a, is denoted by $f^{-1}(x) = \log_a x$.

Namely,

$$\log_a x = y \qquad \Leftrightarrow \qquad a^y = x.$$

In other words, if x > 0 then $\log_a(x)$ is the exponent to which the base a must be raised to give x.

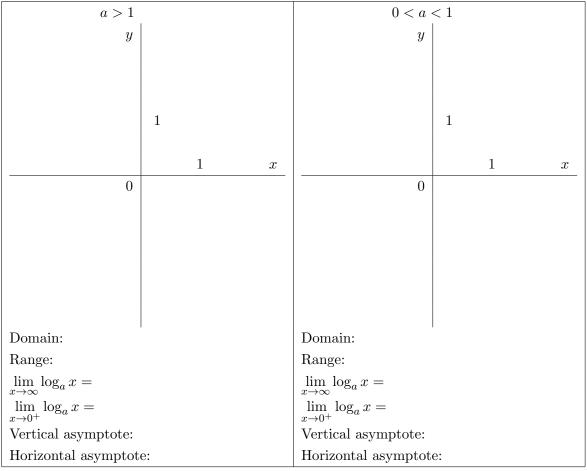
EXAMPLE 2. Evaluate

- (a) $\log_2 16$
- (b) $\log_3 \frac{1}{81}$
- (c) $\log_{125} 5$

CANCELLATION RULES:

- $\log_a a^x = x$ for all $x \in \mathbb{R}$
- $a^{\log_a x} = x$ for x > 0.

Graphs of logarithmic functions $y = \log_a x$:



Properties: Assume that $a \neq 1$ and x, y > 0.

$$\log_a(xy) = \log_a x + \log_a y$$
$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$
$$\log_a(x^y) = y \log_a x$$

Notation: Common Logarithm: $\log x = \log_{10} x$. (Thus, $\log x = y \iff 10^y = x$.) Natural Logarithm: $\ln(x) = \log_e(x)$. (Thus, $\ln x = y \iff e^y = x$.)

Properties of the natural logarithms:

- $\ln(e^x) =$
- $e^{\ln x} =$
- $\ln e =$
- $\log_a x = \frac{\ln x}{\ln a}$, where a > 0 and $a \neq 1$;
- $\lim_{x\to\infty} \ln x =$
- $\lim_{x\to 0^+} \ln x =$

EXAMPLE 3. Find each limit:

(a)
$$\lim_{x \to \infty} \ln(x^2 - x) =$$

(b) $\lim_{x \to 13^+} \log_{13}(x-13) =$

(c) $\lim_{x\to 0^+} \log(\sin x) =$

(d)
$$\lim_{x \to 1} (\ln x)^{\sin x} =$$

EXAMPLE 4. Find the domain of $f(x) = \ln(x^3 - x)$.

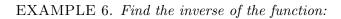
EXAMPLE 5. Solve the following equations:

(a) $\log_{0.5}(\log(x+120)) = -1$

(b) $e^{5+2x} = 4$

(c) $\log(x-1) + \log(x+1) = \log 15$

(d)
$$\ln x^2 - 2\ln \sqrt{x^2 + 1} = 1$$



(a)
$$f(x) = \ln(x+12)$$

(b)
$$f(x) = \frac{10^x - 1}{10^x + 1}$$

EXAMPLE 7. Find an equation of the tangent to the curve $y = e^{2x}$ that is perpendicular to the line x + 2y = 20.

Change of Base formula:

$$\log_a x = \frac{\log_b x}{\log_b a}.$$

In particular,

$$\log_a x = \frac{\ln x}{\ln a}.$$

EXAMPLE 8. Using calculator evaluate $\log_2 15$ to 4 decimal places.