4.4: Derivatives of Logarithmic Functions

EXAMPLE 1. *Using Implicit Differentiation find the derivatives of the following function:*

(a) $f(x) = \ln x$

(b) $f(x) = a^x$

Combining the formulas obtained in Example 1 and Chain Rule one can get

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln(g(x)) =$$

and

$$\frac{\mathrm{d}}{\mathrm{d}x}a^{g(x)} =$$

EXAMPLE 2. *Find the derivative:*

(a) $f(x) = \ln(\sin x)$

(b) $f(x) = \ln(1 + \ln(1 + \ln x))$

(c) $f(x) = \ln|x|$

(d) $f(x) = \cot^3 x + 3^{\cot x}$

EXAMPLE 3. *Using the change of base formula, find the derivative formula for* $f(x) = \log_a x$ *and generalize it using the Chain Rule.*

EXAMPLE 4. *Find the derivative of* $f(x) = \log_2(3 + x^2 + x^3)$

Logarithmic Differentiation can be used to find derivative of complicated functions involving products, quotients or powers.

STEPS IN LOGARITHMIC DIFFERENTIATION:

1. Take logarithms of both sides of an equation $y = f(x)$ and simplify (f.ex. split a product or quotient, etc.).
2. Differentiate implicitly with respect to x.
3. Solve the resulting equation for y'.
4. Plug in $y = f(x)$.

EXAMPLE 5. *Find the derivative:* $y = (\cos x)^{\sin x}$