4.6: Inverse trigonometric functions

- **INVERSE SINE:** If $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then $f(x) = \sin x$ is one-to-one, thus the inverse exists, denoted by $\sin^{-1}(x)$ or $\arcsin x$.

	$y = \sin x$	$y = \arcsin x$
Domain		
Range		

Cancellation equations:

$$\arcsin(\sin x) = x \quad \text{if} \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

and

$$\sin(\arcsin x) = x \quad \text{if} \quad -1 \le x \le 1.$$

EXAMPLE 1. *Find the exact values of the expression:*

(a) $\sin^{-1} 0$

(b) $\arcsin(-1)$

(c) $\sin^{-1}(0.5)$

(d) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$

(e) $\sin\left(\arcsin\frac{2}{7}\right)$

(f) $\tan\arcsin\frac{2}{5}$

(g) $\arcsin\left(\sin\dfrac{5\pi}{4}\right)$

(h) $\arcsin\left(\sin\left(-\dfrac{\pi}{3}\right)\right)$

(i) $\arcsin\left(\sin\dfrac{\pi}{150}\right)$

EXAMPLE 2. *Sketch the graph of* $\arcsin(x)$.

- **INVERSE COSINE:** If $0 \leq x \leq \pi$, then $f(x) = \cos x$ is one-to-one, thus the inverse exists, denoted by $\cos^{-1}(x)$ or $\arccos x$.

	$y = \cos x$	$y = \arccos x$
Domain		
Range		

Cancellation equations:

$$\arccos(\cos x) = x \quad \text{if} \quad 0 \leq x \leq \pi$$

and

$$\cos(\arccos x) = x \quad \text{if} \quad -1 \leq x \leq 1.$$

EXAMPLE 3. *Sketch the graph of* $\arccos(x)$.

EXAMPLE 4. *Find the exact values of the expression:*

(a) $\arccos 0$

(b) $\cos^{-1} 1$

(c) $\arccos(-1)$

(d) $\arccos 0.5$

(e) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$

(f) $\sin\left(2\arccos\frac{3}{5}\right)$

(g) $\arccos\left(\cos\left(\frac{\pi}{6}\right)\right)$

(h) $\arccos\left(\cos\frac{7\pi}{6}\right)$

(i) $\cos\left(\arccos 2\right)$

(j) $\arccos\left(\cos\left(-\frac{\pi}{3}\right)\right)$

- **INVERSE TANGENT:** If $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then $f(x) = \tan x$ is one-to-one, thus the inverse exists, denoted by $\tan^{-1}(x)$ or $\arctan x$.

	$y = \tan x$	$y = \arctan x$
Domain		
Range		

Cancellation equations:

$$\arctan(\tan x) = x \quad \text{if} \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

and

$$\tan(\arctan x) = x \quad \text{for all} \quad x.$$

EXAMPLE 5. *Find the exact values of the expression:*

(a) $\arctan 0$

(b) $\arctan(-1)$

(c) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(d) $\tan(\arccos x)$

(e) $\arctan\left(\tan \frac{5\pi}{4}\right)$

EXAMPLE 6. *Find the following limits:*

(a) $\lim_{x\to\infty} \arctan x =$

(b) $\lim_{x\to-\infty} \arctan x =$

EXAMPLE 7. *Sketch the graph of* $\arctan(x)$.

Derivatives of Inverse Trigonometric Functions:

EXAMPLE 8. **(a)** *Find the derivative of* $f(x) = \arcsin x$.

(b) *Find* $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{\arcsin(3x+1)}\right) =$

TABLE OF DERIVATIVES OF INVERSE TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$$
$$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}, \quad -1 < x < 1$$
$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

EXAMPLE 9. *Find the derivative of* $f(x) = \sin^{-1}(\arctan x)$

EXAMPLE 10. *Find domain of the following functions:*

(a) $f(x) = \arcsin(4x+2)$

(b) $f(x) = \arctan(4x+2)$