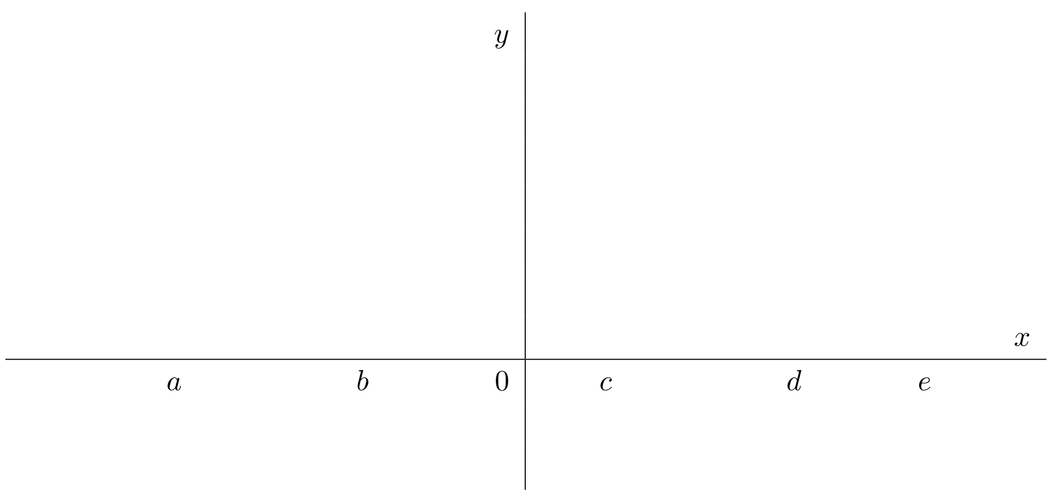
5.2: Maximum and Minimum Values

DEFINITION 1. *Let D be the domain of a function f.*

- *A function f has an* **absolute maximum** *(or* **global maximum***) at $x = c$ if $f(c) \geq f(x)$ for all x in D. In this case, we call $f(c)$ the* **maximum value.**
- *A function f has an* **absolute minimum** *(or* **global minimum***) at $x = c$ if $f(c) \leq f(x)$ for all x in D. In this case, we call $f(c)$ the* **minimum value.**

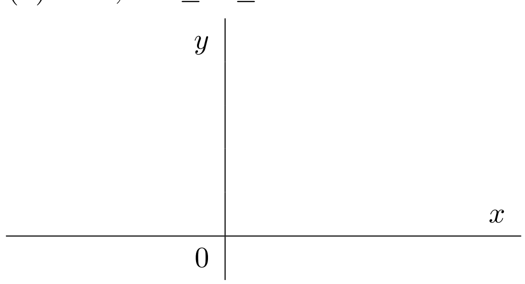
  The maximum and minimum values of f on D are called the **extreme values** *of f.*

DEFINITION 2. *A function f has a local maximum at $x = c$ if $f(c) \geq f(x)$ when x is near c (i.e. in a neighborhood of c). A function f has a local minimum at $x = c$ if $f(c) \leq f(x)$ when x is near c.*



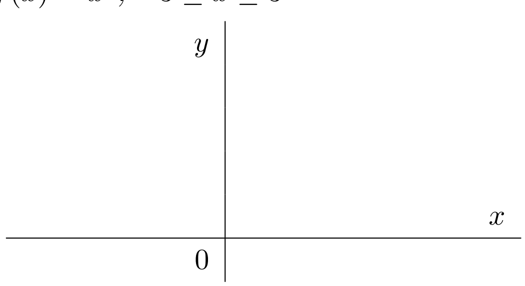
EXAMPLE 3. *Find the absolute and local extrema of f by sketching its graph:*

(a) $f(x) = x^2$, $-1 \leq x \leq 3$



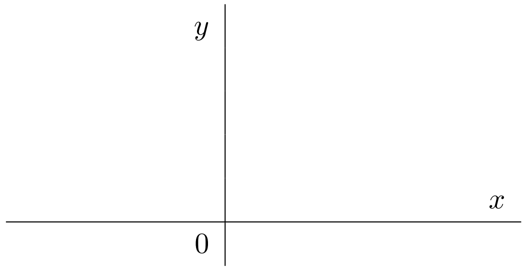
	Local	*Absolute*	*Value*
Maximum			
Minimum			

(b) $f(x) = x^2$, $-3 \leq x \leq 3$



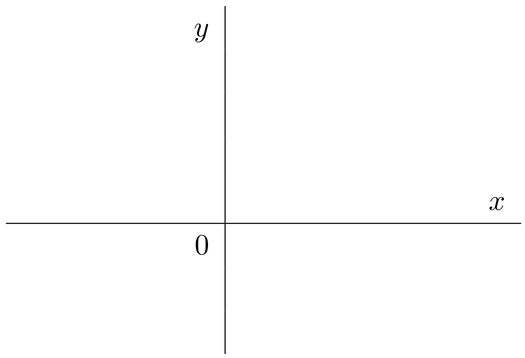
	Local	*Absolute*	*Value*
Maximum			
Minimum			

(c) $f(x) = x^2$



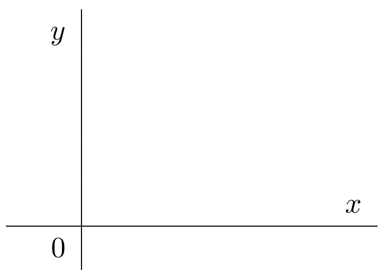
	Local	*Absolute*	*Value*
Maximum			
Minimum			

(d) $f(x) = x^3$



	Local	*Absolute*	*Value*
Maximum			
Minimum			

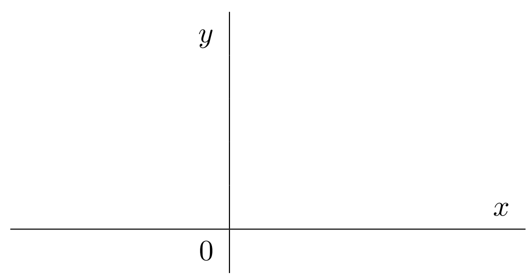
(e) $f(x) = \dfrac{1}{x}$, $0 < x \leq 3$



	Local	*Absolute*	*Value*
Maximum			
Minimum			

(f) $f(x) = \begin{cases} x^4 & \text{if } -1 \leq x < 0 \\ 2 - x^4 & \text{if } 0 \leq x \leq 1 \end{cases}$

	Local	*Absolute*	*Value*
Maximum			
Minimum			



Extreme Value Theorem: If f is a continuous function on a closed interval $[a, b]$, then f attains both an absolute maximum and an absolute minimum.

EXAMPLE 4. *Graph an example of a continuous function on a non closed interval that does not attain an an absolute minimum but does attain an absolute maximum.*

EXAMPLE 5. *Graph an example of a function that is not continuous at a point in the given interval and yet has both absolute extrema.*

DEFINITION 6. *A* **critical number** *of f(x) is a number c is in the domain of f such that either* $f'(c) = 0$ *or* $f'(c)$ *does not exist.*

Illustration:

EXAMPLE 7. *Find the critical numbers of* $f(x)$*:*

(a) $f(x) = x^3 - 3x^2 + 3x$

(b) $f(x) = \left|4 - x^2\right|$

(c) $f(x) = x^{2/5}(5 - x)$

(d) $f(x) = x \ln x$

EXAMPLE 8. *Find the absolute extrema for* $f(x)$ *on the interval* I *where*

(a) $f(x) = x^3 - 3x^2 + 3x,\ I = [-1, 3]$

(b) $f(x) = \sqrt{3}x^2 + 2\cos x^2,\ I = \left[\frac{\sqrt{\pi}}{2}, \sqrt{\pi}\right]$