

## 6.1: Sigma notation

DEFINITION 1. If  $a_m, a_{m+1}, a_{m+2}, \dots, a_n$  are real numbers and  $m$  and  $n$  are integers such that  $m \leq n$ , then

$$\sum_{i=m}^n a_i = a_m + a_{m+1} + a_{m+2} + \dots + a_n.$$

EXAMPLE 2. Compute these summations:

(a)  $\sum_{i=5}^8 (\sin i - \sin(i+1))$

(b)  $\sum_{i=1}^{10} 3$

EXAMPLE 3. Write the sum in sigma notation:

$$-1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25}$$

THEOREM 4. If  $c$  is any constant then

$$\begin{aligned} \sum_{i=m}^n ca_i &= c \sum_{i=m}^n a_i \\ \sum_{i=m}^n (a_i + b_i) &= \sum_{i=m}^n a_i + \sum_{i=m}^n b_i \\ \sum_{i=m}^n (a_i - b_i) &= \sum_{i=m}^n a_i - \sum_{i=m}^n b_i \end{aligned}$$

Note that in general

$$\sum_{i=m}^n a_i b_i \neq \left( \sum_{i=m}^n a_i \right) \cdot \left( \sum_{i=m}^n b_i \right).$$

EXAMPLE 5. If  $\sum_{i=1}^{25} f(i) = 15$ ,  $f(25) = 7$  and  $\sum_{i=1}^{24} g(i) = 25$  find

$$\sum_{i=1}^{24} (2f(i) - g(i))$$

THEOREM 6.

- $\sum_{i=1}^n 1 = n$
- $\sum_{i=1}^n c = nc$ , where  $c$  is a constant.
- $\sum_{i=1}^n i = \frac{n(n+1)}{2}$
- $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^n i^3 = \left[ \frac{n(n+1)}{2} \right]^2$

EXAMPLE 7. Compute these sums:

$$(a) \sum_{i=4}^{27} 20 =$$

$$(b) \sum_{i=1}^n i(i+4) =$$

$$(c) \sum_{k=1}^{100} \left( \frac{1}{k} - \frac{1}{k+1} \right) =$$

EXAMPLE 8. Find the limit:  $\lim_{n \rightarrow \infty} \sum_{j=1}^n \frac{1}{n} \left[ \left( \frac{j}{n} \right)^3 + 1 \right]$