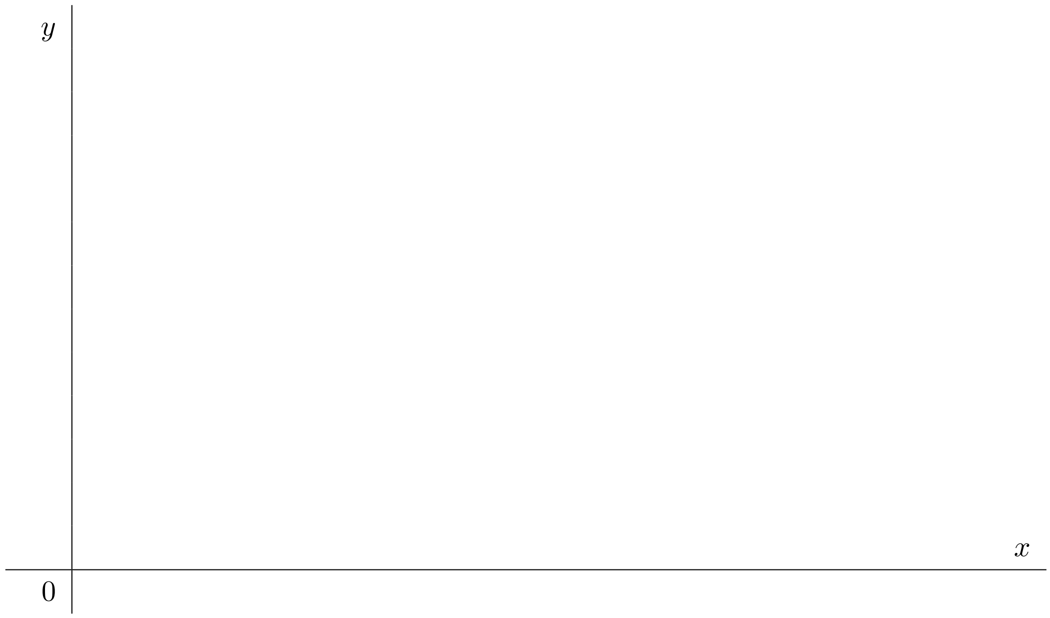
# 6.2: Area

**Area problem**: Let a function $f(x)$ be positive on some interval $[a, b]$. Determine the area of the region between the function and the $x$-axis.




**Solution:** Choose **partition** points $x_0, x_2, \ldots, x_{n-1}, x_n$ so that

$$a = x_0 \leq x_2 \leq \ldots \leq x_{n-1} \leq x_n = b.$$

Use notation $\Delta x_i = x_i - x_{i-1}$ for the length of $i$th subinterval $[x_{i-1}, x_i]$ $(1 \leq 1 \leq n)$
The length of the longest subinterval is denoted by $\|P\|$.
The location in each subinterval where we compute the height is denoted by $x_i^*$.
The area of the $i$th rectangle is

$$A_i =$$

Then

$$A \approx$$

The area $A$ of the region is:

$$A =$$

EXAMPLE 1. *Given* $f(x) = 100 - x^2$ *on* $[0, 10]$. *Let* $P = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ *and* $x_i^*$ *be left endpoint of* $i$*th subinterval.*

**(a)** *Find* $\|P\|$.

**(b )** *Find the sum of the areas of the approximating rectangles.*

**(c)** *Sketch the graph of* $f$ *and the approximating rectangles.*

**Riemann Sum** for a function $f(x)$ on the interval $[a,b]$ is a sum of the form:

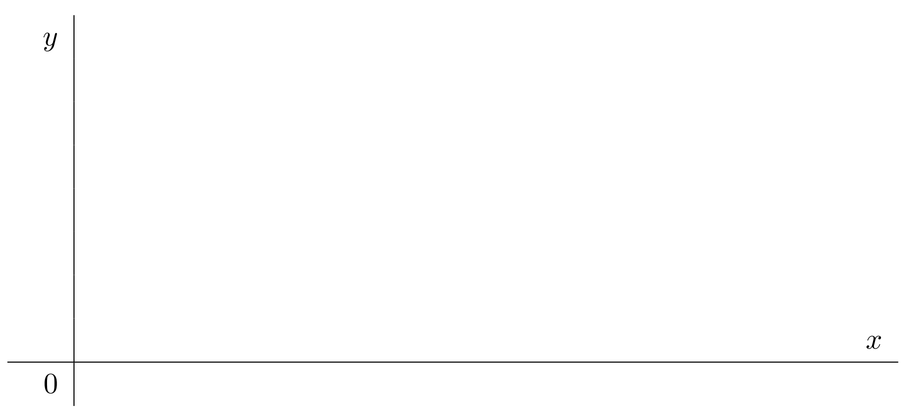
$$\sum_{i=1}^{n} f(x_i^*)\Delta x_i.$$

Consider a partition has equal subintervals: $x_i = a + i\Delta x$, where $\Delta x = \dfrac{b-a}{n}$.

LEFT-HAND RIEMANN SUM : $L_n = \sum_{i=1}^{n} f(x_{i-1})\Delta x = \sum_{i=1}^{n} f(a+(i-1)\Delta x)\Delta x$


RIGHT-HAND RIEMANN SUM : $R_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} f(a+i\Delta x)\Delta x$


MIDPOINT RIEMANN SUM : $M_n = \sum_{i=1}^{n} f\left(\dfrac{x_i + x_{i-1}}{2}\right)\Delta x =$




EXAMPLE 2. *Given* $f(x) = \dfrac{1}{x}$ *on* $[1,2]$. *Calculate* $L_2, R_2, M_2$.

EXAMPLE 3. *Represent area bounded by $f(x)$ on the given interval using Riemann sum. Do not evaluate the limit.*

**(a)** $f(x) = x^2 + 2$ *on* $[0, 3]$ *using right endpoints.*

**(b)** $f(x) = \sqrt{x^2 + 2}$ *on* $[0, 3]$ *using left endpoints.*

EXAMPLE 4. *The following limits represent the area under the graph of $f(x)$ on an interval $[a, b]$. Find $f(x), a, b$.*

**(a)** $\lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \sqrt{1 + \frac{3i}{n}}$

**(b)** $\lim_{n\to\infty} \frac{10}{n} \sum_{i=1}^{n} \frac{1}{1 + \left(7 + \frac{10i}{n}\right)^3}$