Math 172 Exam 1 Review

Do the following problems the textbook: *Section 8.3 # 1,5,7,13,17*

1. If $F(x) = \int_0^{e^x} \cos(t^2)\,dt$ what is $F'(x)$?

2. Find the area between the curves $y = 3x^2$ and $y = 6x$ from $x = 0$ to $x = 4$.

3. Compute $\int_0^1 \frac{2x}{\sqrt[4]{x^2+1}}\,dx$.

4. After an appropriate substitution, the integral $\int_{-1}^{2} 3x^5\sqrt{x^3+1}\,dx$ is equivalent to which of the following?

   (a) $\int_0^3 \left(u^{3/2} - u^{1/2}\right)\,du$ **(b)** $\int_{-1}^2 \left(u^{1/2} - u^{3/2}\right)\,du$ **(c)** $\int_0^3 \left(u^{1/2} + u^{3/2}\right)\,du$

   (d) $\int_0^9 \left(u^{3/2} - u^{1/2}\right)\,du$ **(e)** $\int_{-1}^2 x^3\,u^{1/2}\,du$

5. Determine the area of the region bounded by the x-axis, the curve $y = x^4$ and tangent line to this curve at the point $(-1, 1)$.

6. The height of a monument is 20 m. A horizontal cross-section at a distance x meters from the top is an isosceles triangle with base $x/4$ meters and height $x/3$ meters. Set up, but *do not evaluate*, an integral for the volume of the monument.

7. Find the volume of the solid whose base is the area enclosed by $y = e^x$ and $y = e^{-x}$ from $[0, 1]$ with cross-sections perpendicular to the x-axis that are equilateral triangles.

8. Find the volume of the solid formed by rotating the region bounded by $x = 0$, $y = \ln(x)$, $y = 0$, and $y = 3$ about the y-axis.

9. Using **cylindrical shells** which of the following integrals gives the volume of the solid formed by rotating the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $x = -2$?

   (a) $2\pi \int_0^1 (x-2)(\sqrt{x} - x^2)\,dx$ **(b)** $2\pi \int_0^1 (x+2)(\sqrt{x} - x^2)\,dx$ **(c)** $\pi \int_0^1 (x^2 - \sqrt{x})^2\,dx$

   (d) $\pi \int_0^1 \left[(y^2-2)^2 - (\sqrt{y}-2)^2\right]\,dy$ **(e)** $\pi \int_0^1 \left(y^2 - \sqrt{y}\right)(y+2)\,dy$

10. A 15-Newton weight is suspended vertically at the end of a 30 m long rope. The rope weighs 6 Newtons. How much work (in Newton-m) is required to pull the weight to the top?

11. If a $25J$ work is required to keep a spring $1m$ beyond its natural length, how much work is done in stretching the spring from $2m$ to $4m$ beyond its natural length?

12. A rectangular swimming pool 20 m long, 10 m wide and 3 m deep is full of water (density = ρ kg/m^3). What is the work required to pump all the water out of the top of the pool? (Leave your answer in terms of density ρ and the gravitational constant g.)

13. A tank of water is a trough 10 feet long and has a vertical cross section in the shape of a semi circle with radius 4 feet, diameter at the top. The tank is filled with water. The water is pumped from a spout at the top of the tank that is $0.5ft$ high. Find the work needed to pump out the water until the water level is 1 ft from the bottom. Just set up the integral (assume that the density of water is 62.5lb/ft^3.)

14. A tank has the shape of an upright circular cone with height $15m$ and radius $3m$. In addition, there is a 1 meter high spout at the top of the cone from which the water exits the tank. If the tank is initially full to a water depth of $9m$, find the work required to pump all of the water out of the spout. Just set up the integral. (Leave your answer in terms of density ρ and the gravitational constant g.)

15. Find the average value of $f(x) = \sin^2 x$ over the interval $\left[0, \frac{\pi}{4}\right]$.

16. Compute the following integrals:

    (a) $I = \int_0^1 (x^2 + 4x - 1)e^x \, \mathrm{d}x.$

    (b) $\int \sqrt{x} \ \ln(x) \, dx$

    (c) $I = \int \frac{x}{3} \sin \frac{x-1}{3} \, \mathrm{d}x$

    (d) $I = \int_0^1 \frac{x^2}{e^{10x}} \, dx.$

    (e) $I = \int \arctan \frac{1}{x} \, \mathrm{d}x$

    (f) $I = \int \sin^5 x \sqrt{\cos x} \, \mathrm{d}x$

    (g) $I = \int \frac{\cos^5(\ln x)}{x} \, \mathrm{d}x$

    (h) $\int \cos^2(3x) \sin^2(3x) \, dx$