Math 172 Exam 3 Review

Do the following problems the textbook: *Section 10.6 # 3,5,6,7,11,19*

1. Find the general solution of the differential equation $ty' + 3y = \cos t, \quad t > 0$, and determine how the solutions behave as $t \to +\infty$.
2. Solve the initial value problem $y' - 5y = te^{4t}, \quad y(0) = a$, where a is an arbitrary real constant.
3. Given a series whose partial sums are given by $s_n = (7n+3)/(n+7)$, find the general term a_n of the series and determine if the series converges or diverges. If it converges, find the sum.
4. Find the sum of the following series or show they are divergent:
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{7+5^n}{10^n}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{8}{(n+1)(n+3)}$
5. Write the repeating decimal $0.\overline{27}$ as a fraction.
6. Use the test for Divergence to determine whether the series diverges.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^5}{3(n^4+3)(n+1)}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{6-e^{-n}}$
7. Determine if the sequence $\{a_n\}_{n=2}^{\infty}$ is decreasing and bounded:
   (a) $a_n = \ln n$
   (b) $a_n = \cos n^2$
   (c) $a_n = e^{-n}$
   (d) $a_n = e^n + 11$
   (e) $a_n = 1 - \frac{1}{n^2}$
8. Determine if the sequence converges or diverges. If converges, find its limit.
   (a) $\displaystyle\left\{\frac{2012+(-1)^n}{n^{2012}}\right\}_{n=1}^{\infty}$
   (b) $\displaystyle\left\{\sqrt{\frac{7n+6n^3+n^2}{(n+3)(n^2+8)}}\right\}_{n=4}^{\infty}$.
9. Assuming that the sequence defined recursively by $a_1 = 1$, $a_{n+1} = \frac{1}{2}\left(a_n + \frac{16}{a_n}\right)$ is convergent, find its limit.
10. For what values of x the series $\displaystyle\sum_{n=0}^{\infty}(4x-3)^{n+3}$ converges? What is the sum of the series?
11. Compute $S = \displaystyle\sum_{n=1}^{\infty}(e^{1/n} - e^{1/(n+1)})$.
12. Which of the following series converges absolutely?

(a) $\sum_{n=1}^{\infty}(-1)^{n+5}$

(b) $\sum_{n=1}^{\infty}\frac{\sin(\pi^3n^2)}{n^2\sqrt{n}}$

(c) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt[4]{n}}$

(d) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\ln n}$

(e) $\sum_{n=1}^{\infty}\frac{n^n}{(n!)^2}$

(f) $\sum_{n=1}^{\infty}\frac{5^n}{\ln(n+1)}$

13. Suppose that the power series $\sum_{n=1}^{\infty}c_n(x-4)^n$ has the radius of convergence 4. Consider the following pair of series:

$$(I)\quad \sum_{n=1}^{\infty}c_n5^n \qquad (II)\quad \sum_{n=1}^{\infty}c_n3^n.$$

Which of the following statements is true?

(a) (I) is convergent, (II)is divergent
(b) Neither series is convergent
(c) Both series are convergent
(d) (I) is divergent, (II) is convergent
(e) no conclusion can be drawn about either series.

14. Show that the series $\sum_{n=2}^{\infty}\frac{\ln n}{n^2}$ converges. Then find un upper bound on the error in using s_{10} to approximate the series. (Note that $\ln 2 > 1/2$.)

15. If we represent $\frac{x^2}{4+9x^2}$ as a power series centered at $a=0$, what is the associated radius of convergence?

16. Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty}\frac{(-2)^n(3x-1)^n}{n}$.

17. Which of the following statements is TRUE?

(a) If $a_n > 0$ for $n \geq 1$ and $\sum_{n=1}^{\infty}(-1)^n a_n$ converges then $\sum_{n=1}^{\infty}a_n$ converges.

(b) If $a_n > 0$ for $n \geq 1$ and $\sum_{n=1}^{\infty}a_n$ converges then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

(c) If $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty}(-1)^n a_n$ converges.

(d) If $a_n > 0$ for $n \geq 1$ and $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \frac{e}{2}$ then $\sum_{n=1}^{\infty}a_n$ converges.

18. The series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2 3^n}$ converges to s. Use the Alternating Series Theorem to estimate $|s - s_6|$.