1. Find the average value of $f(x) = \sin x$ on the interval $[0, \pi]$.

(a) $\frac{1}{\pi}$ (b) $\frac{2}{\pi}$ (c) 2 (d) 1 (e) 0

2. Evaluate $\int_0^1 xe^x \, dx$

(a) e (b) $\frac{e}{2}$ (c) 0 (d) $e - 1$ (e) 1

3. Evaluate $\int_0^1 \frac{1}{(x+1)(x+2)} dx$

(a) $-\frac{1}{3}$ (b) $\ln 2$ (c) $\ln\left(\frac{4}{3}\right)$ (d) 3 (e) $\tan^{-1}(2)$

4. The region bounded by the curves $y = x$, $y = 0$, and $x = 1$ is rotated about the vertical line $x = 2$. Find the volume generated.

(a) 2π (b) $\frac{8\pi}{3}$ (c) $\frac{4\pi}{3}$ (d) $\frac{7\pi}{8}$ (e) $\frac{22\pi}{3}$

5. Evaluate $\int_3^8 \frac{3x}{\sqrt{x+1}}\,dx$.

(a) 32 (b) $3\sqrt{8} - 3\sqrt{3}$ (c) 7 (d) 62 (e) 12

6. When a spring of natural length 5 m. is extended to 6 m., the force required to hold it in position is 10N. Find the work done (in Joules) when the spring is extended from 6 m. long to 7 m. long.

(a) 65 (b) 12 (c) 10 (d) 15 (e) 20

7. Find the area bounded by the curves $y = x^2 + 2$ and $y = 3x$ from $x = 0$ to $x = 2$.

(a) 1 (b) 0 (c) $\frac{3}{2}$ (d) $\frac{1}{3}$ (e) $\frac{1}{6}$

8. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx$

(a) 0 (b) $\frac{2}{15}$ (c) $\frac{4}{5}$ (d) 1 (e) $\frac{5}{3}$

9. Suppose that f is a continuous function defined on the interval $[0, 1]$. Given below is a table of values of f:

x	0	1/4	1/2	3/4	1
$f(x)$	1	1	0	$-1/2$	2

Use the data above, along with the *Trapezoidal Rule* with $n = 4$, to compute an approximate value of $\int_0^1 f(x)\,dx$.

(a) 1/4

(b) 1/2

(c) 0

(d) 1/8

(e) 1

10. The region bounded by the curves $y = x^2$ and $y = 2x$ is rotated about the x-axis. Find the volume generated.

(a) 12π (b) $\frac{64\pi}{15}$ (c) $\frac{4}{3}\pi$ (d) 2π (e) $\frac{16\pi}{15}$

11. The triangle bounded by the straight lines $y = 0$, $y = 4x$ and $y + 2x = 6$ is rotated about the x-axis. Set up, but do not evaluate, integrals which give the volume generated using
a) the disk/washer method, b) the cylindrical shells method.

12. A tank is constructed by rotating about the y-axis that part of $y = x^2$ which lies below the horizontal line $y = 4$ (units are feet). The tank is then filled with a liquid weighing 30 lb/ft^3. Find the work done in pumping out the tank.

13. Which of these integrals represents the arc length of $y = x^3$ from $x = 0$ to $x = 1$?

a) $\int_0^1 \sqrt{1 + x^6}\, dx$

b) $\int_0^1 \sqrt{1 + 3x^2}\, dx$

c) $\int_0^1 \sqrt{1 + 9x^4}\, dx$

d) $\int_0^1 2\pi x^3 \sqrt{1 + 9x^4}\, dx$

e) $\int_0^1 \sqrt{1 + x^3}\, dx$

14. By comparing the functions $\dfrac{1}{1+x^3}$ and $\dfrac{1}{x^3}$, what conclusion can be drawn about $\displaystyle\int_1^{\infty} \frac{1}{1+x^3}dx$

a) No conclusion is possible

b) It converges

c) It does not converge

d) Its value is 1/2

e) Its value is 1

15. Does $\displaystyle\int_0^1 \frac{1+x}{\sqrt{x}}\,dx$ converge?

a) YES b) NO

16. When the curve $y = e^x$ from $x = 0$ to $x = 1$ is rotated about the y-axis, which integral represents the surface area?

a) $\int_0^1 2\pi\sqrt{1+e^{2x}}\,dx$

b) $\int_0^1 2\pi e^x\sqrt{1+e^{2x}}\,dx$

c) $\int_0^1 2\pi x\sqrt{1+e^x}\,dx$

d) $\int_0^1 2\pi e^x\sqrt{1+e^x}\,dx$

e) $\int_0^1 2\pi x\sqrt{1+e^{2x}}\,dx$

17. Which of the statements about convergence of $\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0,$ are true?

(1) If $\lim_{n\to\infty} a_n = 0$ then the series converges,

(2) If $a_n \geq \frac{1}{n^2}$ then the series converges,

(3) If $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1$ then the series diverges,

(4) If $a_n \leq \frac{1}{n}$ then the series converges.

(a) (4) only

(b) All

(c) None

(d) (1) only

(e) (2) and (3) only

None are true.(c)

18. The interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}$ is

(a) $(-1, 1]$

(b) $[-1, 1]$

(c) $(-\infty, \infty)$

(d) $[-1, 1)$

(e) $(-1, 1)$

19. Given that $\frac{dy}{dx} = xy$, and $y(0) = 1$, determine y.

(a) $y = e^x$
(b) $y = e^{x^2}$
(c) $y = e^{x^2/2}$
(d) $y = e^{2x}$
(e) $y = e^{2x^2}$

20. The series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(a) Diverges because $a_n \to \infty$

(b) Converges by the ratio test

(c) Diverges by the ratio test

(d) Diverges by the comparison test

(e) Diverges by the integral test

21. The area bounded by the curves $y = 2x$ and $y = \sqrt{x}$ is

(A) $\pi \int_0^4 (\sqrt{x} - 2x)\, dx$

(B) $\int_0^{1/4} (\sqrt{x} - 2x)\, dx$

(C) $\pi \int_0^1 (2x - \sqrt{x})^2\, dx$

(D) $\int_0^4 (2x - \sqrt{x})\, dx$

(E) $2\pi \int_0^4 (2x - \sqrt{x})\, x\, dx$

22. Find an integrating factor $I(x)$ for the differential equation $y' + (\tan^2 x)y = x$, in the interval $0 < x < \pi/2$.

(a) $I(x) = \sec^2 x$
(b) $I(x) = e^{\sec^2 x}$
(c) $I(x) = \tan x - x$
(d) $I(x) = e^{\tan x - x}$
(e) $I(x) = e^{2 \tan x \sec^2 x}$

23. Find the average value of the function $f(x) = \cos^3 x$ on the interval $\left[0, \frac{\pi}{2}\right]$.

A) $\frac{4}{3\pi}$

B) $\frac{3}{\pi}$

C) $\frac{1}{2}$

D) $\frac{\pi}{2}$

E) $\frac{1}{3}$

24. Evaluate $\int x^2 \sin(3x)\,dx$.

25. The improper integral $\int_2^\infty \frac{2+\cos x}{x^4}\,dx$

(A) diverges to $+\infty$.

(B) diverges, but does not approach ∞ because the integrand oscillates.

(C) converges, by comparison with the integral $\int_2^\infty \frac{3}{x^4}\,dx$.

(D) converges to the value $\frac{1}{12}$.

(E) converges, because the integrand oscillates.

26. What integral represents the arc length of the parametric curve segment

$$x = 1 + \cos(2t), \quad y = t - \sin(2t), \quad 0 \le t \le \pi\,?$$

(A) $\int_0^\pi \sqrt{2 - 4\cos(2t) + 4\cos^2(2t)}\, dt$

(B) $\int_0^\pi \sqrt{6 - 4\cos(2t)}\, dt$

(C) $\int_0^\pi \sqrt{2 + 2\cos(2t) + t^2 - 2\sin(2t)}\, dt$

(D) $\int_0^\pi \sqrt{5 - 4\cos(2t)}\, dt$

(E) $\int_0^\pi \sqrt{3 + t^2 + 2\cos(2t) - 2\sin(2t)}\, dt$

27. The integral $\displaystyle\int_0^{\infty} \frac{dx}{(x-2)^2}$

(A) diverges, because of the behavior of the integrand at infinity.

(B) diverges, because of the behavior of the integrand at zero.

(C) converges, by comparison with the integral $\displaystyle\int_1^{\infty} \frac{dx}{x^2}$.

(D) converges, because the integrand approaches a finite constant as $x \to 0$.

(E) none of these.

28. Evaluate $\displaystyle\int \frac{x^2+1}{x^3+2x^2+x}\,dx\,.$

29. Compute the sum of the infinite series $\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+4)}$.

(a)1/5

(b) 1/4

(c) 1/3

(d) 1

(e) 12

30. Compute $\int_{-1}^{1} \frac{1}{x^6}\,dx$.

a. 0

b. $\frac{2}{5}$

c. $-\frac{2}{7}$

d. $-\frac{2}{5}$

e. Divergent

31. Which of the following series are convergent?

(i) $\sum_{n=1}^{\infty} \frac{100^n}{n!}$ (ii) $\sum_{n=1}^{\infty} \frac{2^n}{n+3^n}$

a. both (i) and (ii)
b. (i) only
c. (ii) only
d. neither

32. Compute $\sum_{n=0}^{\infty} \frac{5^{n+1}}{4^n}$.

a. -20

b. $\frac{20}{9}$

c. 20

d. 25

e. divergent

33. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

a. is absolutely convergent.

b. is convergent but not absolutely convergent.

c. is divergent to $+\infty$.

d. is divergent to $-\infty$.

e. is divergent but not to $\pm\infty$.

34. Which of the following is the Maclaurin series expansion of the function $f(x) = \cos(x^2)$?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n+2}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+2}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{4n}$

35. Find the Taylor series for $f(x) = x^2 + 3$ about $x = 2$.

a. $7 + 4(x - 2) + (x - 2)^2$

b. $7 + 4(x - 2) + 2(x - 2)^2 + 4(x - 2)^3$

c. $7 + 4(x - 2) + (x - 2)^2 + \frac{2}{3}(x - 2)^4 + \cdots$

d. $7 + 4(x - 2) + 2(x - 2)^2 + 4(x - 2)^3 + 2(x - 2)^4 + \cdots$

e. $7 + 4(x - 2) + 2(x - 2)^2 + \frac{2}{3}(x - 2)^3 + \frac{4}{3}(x - 2)^4$

36. Find a power series centered at $x = 0$ for the function $f(x) = \frac{x}{1 - 8x^3}$, and determine its radius of convergence.

a. $\sum_{n=0}^{\infty} (-1)^n 8^n x^{3n+1}$ $R = \frac{1}{8}$

b. $\sum_{n=0}^{\infty} (-1)^n 8^n x^{3n+1}$ $R = 8$

c. $\sum_{n=0}^{\infty} \frac{8^n}{n!} x^{3n+1}$ $R = 2$

d. $\sum_{n=0}^{\infty} 8^n x^{3n+1}$ $R = \frac{1}{8}$

e. $\sum_{n=0}^{\infty} 8^n x^{3n+1}$ $R = \frac{1}{2}$

37. ([redacted]) A 50 foot rope that weighs 25 pounds hangs from the top of a tall building. How much work is required pull 10 feet of the rope to the top?

(a) 25 foot pounds

(b) 900 foot pounds

(c) 100 foot pounds

(d) 225 foot pounds

(e) 120 foot pounds

38. Evaluate the integral $\int_0^{1/2} \frac{1}{1+x^3}\,dx$ as an infinite series.

a. $\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{2}\right)^{3n} = 1 - \frac{1}{2^3} + \frac{1}{2^6} - \frac{1}{2^9} + \cdots$

b. $\sum_{n=0}^{\infty}\frac{1}{3n+1}\left(\frac{1}{2}\right)^{3n+1} = \frac{1}{2} + \frac{1}{4 \bullet 2^4} + \frac{1}{7 \bullet 2^7} + \frac{1}{10 \bullet 2^{10}} + \cdots$

c. $\sum_{n=0}^{\infty}\frac{(-1)^n}{3n+1}\left(\frac{1}{2}\right)^{3n+1} = \frac{1}{2} - \frac{1}{4 \bullet 2^4} + \frac{1}{7 \bullet 2^7} - \frac{1}{10 \bullet 2^{10}} + \cdots$

d. $\sum_{n=0}^{\infty}(-1)^n(3n-1)\left(\frac{1}{2}\right)^{3n-1} = -2 - \frac{2}{2^2} + \frac{5}{2^5} - \frac{8}{2^8} + \cdots$

e. $\sum_{n=0}^{\infty}\frac{(-1)^n}{3n-1}\left(\frac{1}{2}\right)^{3n-1} = -2 - \frac{1}{2 \bullet 2^2} + \frac{1}{5 \bullet 2^5} - \frac{1}{8 \bullet 2^8} + \cdots$

39. Let $f(x) = \ln x$.

a) Find the 3^{rd} degree Taylor polynomial T_3 for $f(x)$ about $x = 2$.

b. If this polynomial T_3 is used to approximate $f(x)$ on the interval $1 \leq x \leq 3$, estimate the maximum error $|R_3|$ in this approximation using Taylor's Inequality.

40. ([redacted] Using the error bound formula $|E_T| \leq \frac{K(b-a)^3}{12n^2}$, where $K = \max|f''(x)|$ for $a \leq x \leq b$, what is the smallest value of n so that the approximation T_n (The trapezoidal rule with n subintervals) to the integral $\int_1^3 \ln x\, dx$ is accurate to within $\frac{1}{2400}$?

(a) $n = 40$

(b) $n = 20$

(c) $n = 60$

(d) $n = 30$

(e) $n = 70$

41. Which statement most accurately describes the convergence or divergence of $\int_1^\infty \frac{x\,dx}{\sqrt{x^5+1}}$?

a) The integral converges because $\frac{x\,dx}{\sqrt{x^5+1}} \leq \frac{1}{x^{5/2}}$ and $\int_1^\infty \frac{dx}{x^{5/2}}$ converges.

b) The integral diverges because $\frac{x\,dx}{\sqrt{x^5+1}} \geq \frac{1}{x^{3/2}}$ and $\int_1^\infty \frac{dx}{x^{3/2}} = \infty$.

c) The integral converges because $\frac{x\,dx}{\sqrt{x^5+1}} \leq \frac{1}{x^4}$ and $\int_1^\infty \frac{dx}{x^4}$ converges.

d) The integral diverges because $\frac{x\,dx}{\sqrt{x^5+1}} \geq \frac{1}{x^4}$ and $\int_1^\infty \frac{dx}{x^4} = \infty$.

e) The integral converges because $\frac{x\,dx}{\sqrt{x^5+1}} \leq \frac{1}{x^{3/2}}$ and $\int_1^\infty \frac{dx}{x^{3/2}}$ converges.

42. Set up the integral that will compute the area of the surface obtained by revolving the curve $x = (y-1)^2$ from $(0,1)$ to $(1,2)$about the y-axis.

a) $\int_1^2 \sqrt{1+4(y-1)^2}\,dy$ b) $\int_0^1 2\pi(x-1)^2\sqrt{1+4(x-1)^2}\,dx$ c) $\int_0^1 \sqrt{1+4(x-1)^2}\,dx$

d) $\int_1^2 2\pi(y-1)^2\sqrt{1+4(y-1)^2}\,dy$ e) $\int_1^2 \pi(y-1)^4\,dy$

43. Given $\frac{du}{dt} = e^{2t-u}$ and $u(0) = 1$, find $u(1)$.

(a) $u(1) = \ln(\frac{1}{2}e^2 + e - \frac{1}{2})$
(b) $u(1) = \ln(e+1)$
(c) $u(1) = \ln(2e^2 + e - 2)$
(d) $u(1) = \ln(\frac{1}{2}e^2 - e + \frac{1}{2})$
(e) $u(1) = \ln(e-1)$