Chapter 1 Review

$\neg(\forall x \in D, P(x)) \equiv$

$\neg(\exists x \in D \ni P(x)) \equiv$

1. Negate the following statements:

   (a) All triangles are isosceles.

   (b) Some odd numbers are multiples of three.

   (c) The absolute value of **the** real number x is less than 2016. ˜

   (d) Some angles of a triangle are greater than $\pi/2$.

   (e) Every prime number is greater than two.

   (f) The sum of an even number and a prime number is odd.

   (g) The square of an even integer is divisible by four.

   (h) There is a real-valued function $f(x)$ such that $f(x)$ is not differentiable at any real number x.

   (i) If x and y are real numbers such that $x^3y^2 = 0$, then $x = 0$ or $y = 0$.

   (j) There exists a rational number r such that $1 < r < 2$.

   (k) Some integers are neither prime nor even.

2. Given a quantified statement

$$\forall n \in \mathbb{O},\ \exists x \in \mathbb{Z} \ni\ n = 4x + 1 \ \vee\ n = 4x + 3. \tag{1}$$

   (a) Express the statement (1) in words.

   (b) Express the **negation** of the statement (1) in symbols. **(Do NOT use the symbol $\notin$.)**

3. Prove:

   ``Let $x, y \in \mathbb{Z}$. Then x and y are of the same parity if and only if $x + y$ is even.''

4. Consider the following statement

   ``Let $a, b \in \mathbb{Z}$. If a is odd and $a + b$ is even, then b is odd.

   (a) Write out the contrapositive statement.

   (b) Prove the given statement by direct proof

   (c) Prove the given statement by contradiction.