12.8: Lagrange Multipliers

PROBLEM: *Maximize/minimize a general function $u = f(x, y, z)$ subject to a constraint (or side condition) of the form $g(x, y, z) = k$.*

SOLUTION:

Note that the equation $g(x, y, z) = k$ represents a ________________ in $\mathbb{R}^3$. Denote this surface by S.

Suppose that f has an extreme value at a point $P(x_0, y_0, z_0)$ on S and let C be a curve with vector equation:

$$C: \quad \mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$$

that lies on S and passes through P.

If t_0 is the parameter value corresponding to the point P then

$r(t_0) =$.

The values that f takes on the curve C:

Since f has an extreme value at $P(x_0, y_0, z_0)$, it follows that

METHOD OF LAGRANGE MULTIPLIERS: *To Maximize/minimize a general function $u = f(x, y, z)$ subject to a constraint of the form $g(x, y, z) = k$ (assuming that these extreme values exist):*

1. *Find all values x, y, z and λ (a Lagrange multiplier) s.t.*

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z)$$

and

$$g(x, y, z) = k$$

2. *Evaluate f at all points (x, y, z) that arise from the previous step. The largest of these values is the* $\max f$*; the smallest is the* $\min f$*.*

Rewrite the system

$$\begin{gathered} \nabla f(x, y, z) = \lambda \nabla g(x, y, z) \\ g(x, y, z) = k \end{gathered}$$

in component form:.

Method of Lagrange Multipliers for function of two variables:

EXAMPLE 1. *Use Lagrange multipliers to solve Example 10 from Section 12.7: Find the points on the surface* $z^2 = xy + 1$ *that are closest to the origin.*

EXAMPLE 2. *Use Lagrange multipliers tofind the maximum and minimum values of* $f(x,y) = x^2 + y^2$ *subject to* $x^4 + y^4 = 1$.