13.5: Double integrals in polar coordinates

EXAMPLE 1. *Evaluate*
$$I = \iint_D \arctan \frac{y}{x} \, \mathrm{d}A$$
where $D = \left\{ (x, y) \mid 1 \le x^2 + y^2 \le 4, x \le y \le \sqrt{3}x, x \ge 0 \right\}.$

THEOREM 2. *Change to polar coordinates in a double integral: Let f be a continuous on the region D. Denote by D^* the region representing D in the polar coordinates (r, θ). Then*

$$\iint_D f(x, y) \, \mathrm{d}A = \iint_{D^*} f(r \cos \theta, r \sin \theta) \, r \, \mathrm{d}r \mathrm{d}\theta.$$

REMARK 3. Be careful not to forget the additional factor r on the right side of the formula.

Solution of Example 1: Evaluate $I = \displaystyle\iint_D \arctan \frac{y}{x}\, \mathrm{d}A$
where $D = \left\{(x, y)|\ 1 \le x^2 + y^2 \le 4,\, x \le y \le \sqrt{3}x,\, x \ge 0\right\}.$

EXAMPLE 4. *Find the volume of the solid that lies under the paraboloid*
$z = x^2 + y^2$, *above the xy-plane and inside the cylinder* $x^2 + y^2 = 2x$.

EXAMPLE 5. *Find the area of the region inside the circle $r = 4\sin\theta$ and outside the circle $r = 2$.*