14.1: Vector Fields

A vector function

$$\mathbf{r} = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

is an example of a function whose domain is a set of real numbers and whose range is a set of vectors in $\mathbb{R}^3$:

$$\mathbf{r}(t) : \mathbb{R} \to \mathbb{R}^3.$$

Consider a type of functions (**vector fields**) whose domain is $\mathbb{R}^2$ (or $\mathbb{R}^3$) and whose range is a set of vectors in $\mathbb{R}^2$ (or $\mathbb{R}^3$):

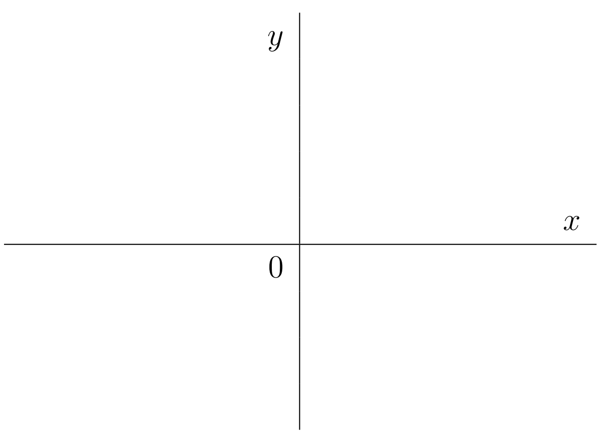
Vector field over $\mathbb{R}^2$.

$$\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j} = \langle P(x, y), Q(x, y)\rangle$$

Vector field over $\mathbb{R}^3$:

$$\mathbf{F}(x, y, z) = P(x, y, z)\mathbf{i} + Q(x, y, z)\mathbf{j} + R(x, y, z)\mathbf{k} = \langle P(x, y, z), Q(x, y, z), R(x, y, z)\rangle$$

EXAMPLE 1. *Describe the vector field* $\mathbf{F}(x, y) = -y\mathbf{i} + x\mathbf{j}$ *by sketching.*



Function $u = f(x, y, z)$ is also called a **scalar field**. Its gradient is also called **gradient vector field**:

$$\mathbf{F}(x, y, z) = \nabla f(x, y, z) =$$

EXAMPLE 2. *Find the gradient vector field of* $f(x, y, z) = xyz$.