

Step 2. Now you need to find the absolute extremum of the function along the boundary of D . First get the values of f at corner points:

The boundary of D is given by the following conditions:

- right side: $x = 1, -1 \leq y \leq 1$
- left side: $x = \boxed{}, -1 \leq y \leq 1$
- upper side: $y = 1, \boxed{}$
- lower side: $\boxed{}, \boxed{}$

Along the right side you know that $x = 1$. Using this, define a new function as follows,

$$g(y) = f(1, y) = \boxed{}$$

Find the critical points of $g(y)$ in the range $-1 < y < 1$ and then evaluate $g(y)$ at the critical points:

$$g'(y) = 8y - 2 = 0 \Rightarrow y = 1/4.$$

This is in the range and so you will need to evaluate $g(\frac{1}{4}) = 4.75$. Notice that, using the definition of $g(y)$ this is also function value for $f(x, y)$:

$$g\left(\frac{1}{4}\right) = f\left(1, \frac{1}{4}\right) = 4.75$$

Now do the left side of D . Notice that, for this boundary, this is the same function as you looked at for the right side: $f(1, y) = f(-1, y)$.² So, we will have: $f(-1, \frac{1}{4}) = 4.75$

Look at the upper side. Define a new function:

$$h(x) = f(x, 1) = \boxed{}.$$

Find the critical points of $h(x)$ in the range $-1 < x < 1$ and then evaluate $h(x)$ at the critical points:

Look at the lower side (Again, define a new function and find its critical point/s in the range $-1 < x < 1$.)

Step 3. Collect up all the function values for that you've computed in this problem:

(x, y)	(0, 0)	(1, -1)	(1, 1)	$(1, \frac{1}{4})$	(-1, 1)	(-1, -1)	$(-1, \frac{1}{4})$	(0, 1)	(0, -1)
$f(x, y)$	4	11	7	4.75	7	11	4.75	8	8

²This will not always happen.

The absolute minimum is at $(0, 0)$ and its value is 4 and the absolute maximum occurs at $(\pm 1, -1)$ and its value is 11.

Example 2 Find the absolute minimum and absolute maximum of $f(x, y) = 2x^2 + y^2 + y + 1$ on the region $D = \{(x, y) | x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}$.

Solution.

Sketch the region D here:



STEP 1. Find all the critical points that lie

inside D . To do this you need the two first order derivatives in order to solve the system

$$f_x = \boxed{} = 0$$

$$f_y = \boxed{} = 0$$

Write its solution here: $\boxed{}$. Recall that we only want critical points in the region that we are given.

You aren't going to be classifying the critical points. Thus, you don't need the Second Order Derivatives. Evaluate the value of f at a critical point only if it belongs to D .

Step 2. Now you need to find the absolute extrema of the function along the boundary of D .

First get the value of f at corner points:

Describe/parametrize the boundary of D .

- left side: $x = 0, 0 \leq y \leq 1$
- lower side: $y = 0, 0 \leq x \leq 1$
- right side (arc of the circle): $x = \sqrt{1 - y^2}, 0 \leq y \leq 1$. (Notice here that there are many other parameterizations that will also work here. For example, you can use $x = \cos t, y = \sin t$ for $0 \leq t \leq \pi/2$.)

Along the left side you know that $x = 0$. Using this, define a new function as follows,

$$g(y) = f(0, y) = \boxed{}$$

