14.1: Vector Fields

A vector function

$$\mathbf{r} = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

is an example of a function whose domain is a set of real numbers and whose range is a set of vectors in \mathbb{R}^3 :

$$\mathbf{r}(t): \mathbb{R} \to \mathbb{R}^3$$
.

Consider a type of functions (**vector fields**) whose domain is \mathbb{R}^2 (or \mathbb{R}^3) and whose range is a set of vectors in \mathbb{R}^2 (or \mathbb{R}^3):

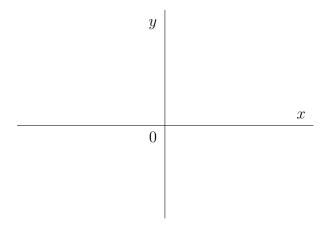
Vector field over \mathbb{R}^2 .

$$\mathbf{F}(x,y) = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j} = \langle P(x,y), Q(x,y) \rangle$$

Vector field over \mathbb{R}^3 :

$$\mathbf{F}(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k} = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$$

EXAMPLE 1. Describe the vector field $\mathbf{F}(x,y) = -y\mathbf{i} + x\mathbf{j}$ by sketching.



Function u = f(x, y, z) is also called a scalar field. Its gradient is also called gradient vector field:

$$\mathbf{F}(x,y,z) = \nabla f(x,y,z) =$$

EXAMPLE 2. Find the gradient vector field of f(x, y, z) = xyz.