# 14.9: The Divergence Theorem

Let $E$ be a simple solid region with the boundary surface $S$ (which is a closed surface.) Let $S$ be positively oriented (i.e.the orientation on $S$ is outward that is, the unit normal vector $\hat{\mathbf{n}}$ is directed outward from $E$).

**The Divergence Theorem**: Let $E$ be a simple solid region whose boundary surface $S$ has positive (outward) orientation. Let $\mathbf{F}$ be a continuous vector field on an open region that contains $E$. Then

$$\iint_S \mathbf{F} \cdot \mathrm{d}\mathbf{S} = \iiint_E \mathrm{div}\mathbf{F}\, \mathrm{d}V.$$

EXAMPLE 1. *Let* $E = \{(x, y, z) : x^2 + y^2 \le R^2, 0 \le z \le H\}$. *Find the flux of the vector field* $\mathbf{F} = \langle 1 + x, 3 + y, z - 10\rangle$ *over* $\partial E$.

REMARK 2. If $\mathbf{F} = \left\langle \frac{x}{3}, \frac{y}{3}, \frac{z}{3} \right\rangle$ then

EXAMPLE 3. *Let $E$ be the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the $xy$-plane. Evaluate* $I = \iint_S \langle x^3, 2xz^2, 3y^2 z\rangle \cdot \mathrm{d}\mathbf{S}$ *if*

**(a)** *$S$ is the boundary of the solid $E$.*

**(B)** *$S$ is the part of the paraboloid $z = 4 - x^2 - y^2$ between the planes $z = 0$ and $z = 4$.*

EXAMPLE 4. *Evaluate* $I = \iint_S \mathrm{curl}\mathbf{F} \cdot \mathrm{d}\mathbf{S}$ *if* $S$ *is the boundary of*

**(a)** *ellipsoid* $E = \left\{ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \right\}$ *and* $\mathbf{F} =$ ;

**(b)** *an arbitrary simple solid region* $E$ *and* $F$ *is an arbitrary continuous vector field.*