# Math 220 – Homework 11

## Due Friday 12/06 at the beginning of the final exam

## PART A

Problems from the textbook:

- Section 6.1 # 2, 3
- Section 6.2 # 1(a)

## PART B

1. Let $a, b \in \mathbb{Z}$ with $a$ and $b$ not both zero. Prove that if $d = \gcd(a, b)$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.
2. Let $a = -255$ and $b = 143$
   (a) Use the Euclidean Algorithm to determine $\gcd(a, b)$.
   (b) Find integers $x$ and $y$ such that $ax + by = \gcd(a, b)$.
3. (a) Write the integer 42750 in a compact standard form.
   (b) Determine the following, representing your answer in the compact standard form:
   $$\gcd((-1)^{2020} \cdot 2^{2019} \cdot 3^4 \cdot 55 \cdot 7^2, (-1)^{2021} \cdot 6 \cdot 3^2 \cdot 77)$$
4. Prove that if $p$ is a prime number greater than 3, then $p$ is of the form $3k + 1$ or $3k + 2$.
5. Prove that if $p$ is a prime number, then $\sqrt[n]{p}$ is irrational for every integer $n \geq 2$.
6. Prove or disprove that 3 is the only prime number of the form $n^2 - 1$.
7. Prove that if $a$ is a positive integer of the form $3n + 2$, then at least one prime divisor of $a$ is of the form $3n + 2$.