

# Math 300 – Homework 1

Due Thursday 9/6 at the beginning of class

Total points: 195 (Writing portion: 100 pts (all the problems marked by \*).)

## PART A\*

Problems from the textbook:

• Section 1.1	problem	2(b)	3(b)	4(d,e)
	points	5	5	10

## PART B

- 10 points For the predicate  $P(x) : (x^3 + 1)(x^2 - 3) = 0$ , where  $x \in \mathcal{U}$ , determine:

  - the values of  $x$  for which  $P(x)$  is a true statement if  $\mathcal{U} = \mathbb{R}$ .
  - the values of  $x$  for which  $P(x)$  is a false statement if  $\mathcal{U} = \mathbb{N}$ .
- 21 points Determine whether each of the following sentences is a proposition, predicate, or neither.

  - $20^2 + 20^2 > 2020^2$
  - $x^2 = -1$ .
  - For every real number  $x$ ,  $x^2 \neq -1$ .
  - The product of every two prime numbers is odd.
  - Schreck is six feet tall.
  - Harry Potter and the Sorcerer's Stone.
  - Give an example of integrable function.
- 20 points State the negation for each of the following propositions.

  - $\sqrt{3}$  is a rational number.
  - 0 is not a negative number.
  - The real number  $r$  is at most  $\sqrt{3}$
  - Two sides of a triangle have the same length.
  - The point  $P$  on the plane lies outside of the circle  $C$ .
- \* 10 points For the predicates  $p(x) : x + 1 \geq 4$  and  $q(x) : 13 < 4x$  over a domain (universe)  $S = \{0, 2, 3, 4, 6\}$ , determine all values of  $x \in S$  for which the biconditional  $P(x) \Leftrightarrow Q(x)$  is true. Show all work.
- 12 points In each of the following statements identify the hypothesis (assumption) and conclusion. Represent your answers in the following form:

**Hypothesis:**

**Conclusion:**

  - If  $a$  is irrational, then  $2a$  is irrational.
  - $a^3$  is an even integer whenever  $a$  is an even integer.
  - In order to pass the drivers test, the candidate must be able to parallel park.
- 32 points Consider the following propositions

$$P : 2018 \in 3\mathbb{Z} \quad \text{and} \quad Q : 3^{2018} \in \mathbb{O}.$$

Write each of the following compound statements in words and indicate whether it is true or false.

- (a)  $P$ ; (b)  $Q$ ; (c)  $\neg P$ ; (d)  $P \vee Q$ ; (e)  $P \wedge Q$ ; (f)  $P \Rightarrow Q$ ; (g)  $\neg Q \Rightarrow P$ ; (e)  $P \Leftrightarrow Q$ .

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**70 points** PART C\*

Rewrite each sentence according to provided guidelines (see “Communicating Mathematics” on eCampus), then using complete sentences give a reason to changes you made.

1. In mathematics, an irrational number  $r$  is a real number that cannot be expressed as a ratio of integers, e.g. as a fraction.
2. Let  $a, b, c, M$ , and  $N$  be given integers.
3.  $m^4 + m^2 + 2018$  is positive for every real  $m$ .
4. Pure mathematics topics often turn out to have applications, i.e. number theory in cryptography.
5. If  $x, y$  are integers of the same parity, then  $x + y$  is even.
6. The square of every integer  $n$  is even.
7.  $f$  is differentiable everywhere.
8. Every number  $\in \mathbb{E}$  is divisible by 2.