

Math 300 – Homework 2

Due Thursday 9/12 at the beginning of class

Total points: 132 (Writing portion: 8 pts (all the problems marked by *).)

PART A

Problems from the textbook:

• Section 1.1	problem	10(a,b)	14(b,c)
	points	10	10

PART B

- [8 points] In each of the following statements identify the hypothesis (assumption) and conclusion.

 - If a is irrational, then $2a$ is irrational.
 - a^3 is an even integer whenever a is an even integer.
- [8 points] Without changing their meanings, convert each of the following sentences into a sentence having the form “If P , then Q .”

 - A function is integrable provided the function is continuous.
 - A function is rational if it is a polynomial.
- [4 points] Without changing its meaning, convert the sentence “If $xy = 0$, then $x = 0$ or $y = 0$, and conversely.” into a sentence having the form “ P if and only if Q .”
- * [8 points] Prove that the statement $\neg((\neg Q \wedge (P \Rightarrow Q)) \Rightarrow (\neg P))$ is a tautology, a contradiction, or neither. You must state which of the three it is as well as give the proof.
- [25 points] Negate the following statements:

 - Every real number is less than 100.
 - There is a politician who is honest or trustworthy.
 - If f is a linear function, then f is continuous at 0.
 - If a differentiable function f has a local minimum at the point x_0 , then $f'(x_0) = 0$.
- Given a quantified statement

$$\forall x \in \mathbb{Z}^+, (\exists y \in \mathbb{Z}^+ \ni xy \in \mathbb{E}). \quad (1)$$

- [3 points] Express the given statement (1) in words.
 - [6 points] Express the **negation** of the given statement (1) in symbols. (**Do NOT use the symbol “ \notin ”.**)
 - [3 points] Express the **negation** of the given statement (1) in words.
- Given a quantified statement

$$\forall x \in \mathbb{R}, \exists n \in \mathbb{Z} \ni (n \leq x < n + 1). \quad (2)$$

- (a) [3 points] Express the statement (2) in words.
- (b) [8 points] Express the **negation** of the statement (2) in symbols. (**Do NOT use the symbol “ \notin ” and interval notation.**)
8. [36 points] Express the following statements in the form “*For all ... , if ... then ...*” using symbols to represent variables. Then write their negations in words, again using symbols to represent variables. (**Attention you should use symbols to represent introduced variables only. The statements and their negations must be written in words and not in symbols.**)
- (a) An integer is odd or even.
- (b) All angles of a square are equal.
- (c) The number -1 is the largest negative integer.
- (d) When the product of two integers is odd, then the both integers are odd.
- (e) Every multiple of 6 is even and is not a multiple of 4.
- (f) The square of an even integer is divisible by 4.