Math 300 – Homework 3

Due Thursday 9/19 at the beginning of class

Total points: 218 (Writing portion: 80 pts (all the problems marked by $*$).)

PART A

Problems from the textbook:

- Section 1.1

problem	15(a,b,e, g, h,i)	16(a,b,c,d,e)
points	30	50

- Section 1.2

problem	1(b)*	2(a,b)*	4*
points	10	20	10

PART B

1. [4 points] Write the following statement using "if, then":

   "A sufficient condition for a triangle to be isosceles is that it has two equal angles."

2. [6 points] Consider the following definition:

   A real-valued function $f(x)$ is said to be **one to one** *if for all $x_1, x_2 \in \mathbb{R}$, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.*

   Write the negation of this definition completing the following: *"A real-valued function $f(x)$ is said to be* **not one to one** *if ..."*

3. Consider the following definition:

   A real-valued function $f(x)$ is said to be **decreasing** *on the closed interval $[a, b]$, if for all $x_1, x_2 \in [a, b]$, if $x_1 < x_2$, then $f(x_1) > f(x_2)$.*

   (a) [6 points] Write the negation of this definition completing the following: *"A real-valued function $f(x)$ is said to be* **not decreasing** *on the closed interval $[a, b]$, if ..."*

   (b) [6 points] Give an example of a function that is decreasing on the interval $[1, 3]$ and based on the above definition explain why your example is correct.

   (c) [6 points] Give an example of a function that is not decreasing on the interval $[1, 3]$ and based on the negation of the above definition explain why your example is correct.

4. [18 points] Let n represents some fixed integer. In each of the following statements identify the hypothesis (assumption) and conclusion.

   (a) The number n divides 5 only if n divides 10,

   (b) The condition $n^2 \in 3\mathbb{Z}$ is necessary for n to be a multiple of 3.

   (c) The condition $n \in \mathbb{E}$ is sufficient for n to be a multiple of 4.

5. [12 points] For the statement
   S: `‘‘For every integer` n, `if` n `is divisible by` 3 `and` n `is divisible by` 5, `then` n `is divisible by` 15.`’’` write in a useful form

   (a) the converse of S;

   (b) the contrapositive of S.

   (c) the converse of the contrapositive of S;

   (d) the contrapositive of the converse of S.

6. * [10 points] Let $x \in \mathbf{R}$. Prove that if $|x| < 1$, then $x^2 - 2x + 2 \neq 0$.

7. * [10 points] Let $x \in \mathbf{R}^+$. Prove that if $x^4 - 2x^2 + 2 \leq 0$, then $x^{2019} \geq 2019$.

8. * [10 points] Prove that if n is an even integer, then $n^{2019} + 19(n-1)^2 - 2019$ is even. (Give a formal proof).

9. * [10 points] Prove that if x and y are odd integers, then $xz + 3yz$ is even for every integer z. (Give a formal proof).