

# Math 300 – Homework 4

Due Thursday 9/26 at the beginning of class

Total points: 190 (Writing portion: 190 pts (all the problems marked by \*).)

## PART A

Problems from the textbook:

• Section 2.1	problem	4*	5(a,b,e,i)*
	points	10	40

## PART B

- \*[50 points] Let  $x, y, z \in \mathbb{Z}$ . Give a formal proof of the following statements.
  - If  $7x + 5$  is odd, then  $x$  is even.
  - If  $xy$  is odd, then  $x$  and  $y$  are odd.
  - If  $3x + 7y$  is even, then  $x$  and  $y$  are of the same parity.
  - If  $x \nmid yz$ , then  $x \nmid y$  and  $x \nmid z$ .
  - The integers  $13x - 11$  and  $17x + 2$  are of opposite parity.
- \* [10 points] Prove that the sum of every three consecutive integers is divisible by 3. (Give a formal proof.)
- \* [40 points] Let  $a$  and  $b$  be integers. Prove the following statements.
  - If  $a|b$ , then  $a|(b^{2018} - b^{2020} + 2019b)$ .
  - If  $a^2|a$ , then  $a \in \{-1, 0, 1\}$ .
  - If  $a|b$  and  $b|a$ , then  $a = b$  or  $a = -b$ .
  - If  $a|b$ , then  $a^2|b^2$ .
- \* [40 points] Prove or disprove the following statements:
  - If  $a, b, c \in \mathbb{R}$ , then  $\sqrt{a^2 + b^2 + c^2} = \sqrt{a^3 + b^3 + c^3}$ .
  - For all positive integers  $x, y$ , and  $z$ , the equality  $x^{y^z} = (x^y)^z$  holds.
  - If  $k \in \{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$ , then  $4^k + 3^{2k} + 4k(k-1)(2k-1)$  is prime.
  - If  $x, y \in \mathbb{R}$ , then  $|x - y| = |x| - |y|$ .