

# Math 300 – Homework 5

Total points: 0

## PART A

Problems from the textbook:

- Section 2.2 # 3, 5
- Section 3.1 # 1, 2, 11, 13.

## PART B

1. Prove that if  $a$  and  $b$  are odd integers, then  $4 \nmid (a^2 + b^2)$ .
2. Prove that if  $x$  is an integer, then  $x^3$  has the same parity as  $x$ .
3. (a) Let  $n \in \mathbb{Z}$ . Prove that if  $2 \mid (n^2 - 5)$ , then  $4 \mid (n^2 - 5)$ .  
(b) Give an example of an integer  $n$  such that  $2 \mid (n^2 - 5)$ , but  $8 \nmid (n^2 - 5)$
4. Consider the statement:  

*“If the product of two integers is even, then at least one of these integers is even.”*

(a) Rewrite the statement in symbols.  
(b) Give a formal proof.
5. Let  $a$  be a positive real number. Prove that there is a unique positive real number  $x$  such that  $x^4 - a^4 = 0$ .
6. Prove by induction that if  $n$  is a positive integer, then  $9^n - 4^n \in 5\mathbb{Z}$ .