

Math 300 – Homework 6

Due Thursday 10/17 at the beginning of class

Total points: 258 (Writing portion: 85 pts (all the problems marked by *).)

PART A

Problems from the textbook:

• Section 4.1	problem	1(a,b,d,g,j)	4*	5*
	points	20	10	10

• Section 4.2	problem	1(a,b,h,f)*	2*	11	15	16(c)	17	18
	points	40	10	5	10	5	5	15

PART B

1. [20 points] Consider the following sets:

$$A = \{n \in \mathbb{R} \mid |n| < 2\}, \quad B = \{n \in \mathbb{Z} \mid n^3 = n\}, \quad C = \{n \in \mathbb{Z} \mid n^2 \leq n\},$$

$$D = \{x \in \mathbb{Z} \mid x^2 \leq x\}, \quad E = \{-1, 0, 1\}, \quad F = \{t \in \mathbb{Z} \mid |t| < 2\}$$

Determine whether the following statements are true or false. justify your answers.

- (a) $A = E$
 (b) $C = D$
 (c) $C \subset B$
 (d) $F \subset A$
2. [10 points] Let A , B and C be nonempty subsets of a universal set U . Draw a Venn diagram for each of the following set operations.
 (a) $(B \cup (A - C)) \cup B$ (b) $A \cap (C - B)$
3. * [15 points] Let A , B , and C be nonempty subsets of a universal set U . Disprove the following statements. (Hint: Venn Diagram might help to find a counterexample).
 (a) If A is not a subset of B and B is not a subset of A , then $A \cap B = \emptyset$.
 (b) If $A \cap B = A \cap C$, then $B = C$.
 (c) If $A - B = C - B$, then implies $A = C$.
4. [9 points] Describe the following sets by listing their elements.
 (a) The set of all remainders when an even integer is divided by 4.
 (b) The set of all odd integers of absolute value less or equal than 7.
5. [12 points] Describe the following sets by listing enough elements to indicate a pattern for all elements of the set.

- (a) The set of all numbers x for which $\tan x$ is undefined.
- (b) $A = \{3q - 2 \mid q \in \mathbb{Z}\}$
6. [20 points] Let $U = \{1, \{2\}, a, b, c\}$ be the universal set and let $A = \{a, b, 1, \{2\}\}$, $B = \{b, c\}$, $C = \{1, \{2\}, a, c\}$. Determine the following (show all intermediate steps):
- (a) $\overline{A} \cup (B \cap C)$
- (b) $\overline{B \cup C} \cap U$
- (c) $\overline{(A \cup B) - (B \cap C)}$
- (d) $(B \cap C) \times A$
7. [10pts] Given $A = \{x \in \mathbb{Z} \mid |x| > 10\}$. Compute the complement of A , if
- (a) $U = \mathbb{Z}$.
- (b) $U = \mathbb{R}$.
8. [12 points] Let $U = \mathbb{R}$ be the universal set. Consider $A = \{x \in \mathbb{R} \mid |x - 1| \geq 4\}$ and $B = \{x \in \mathbb{R} \mid \left|\frac{x}{2} - 20\right| > 1\}$.
- (a) Express the sets A and B using interval notation (as an interval or a union of intervals).
- (b) Determine $A \cap \overline{B}$ as an interval or a union of intervals.
9. [20 points] Assume that A , B and C are sets and P , Q and R are propositions. Characterize the following expressions as either
- (i) a proposition/statement
- (ii) not a proposition/statement, but an expression that makes sense mathematically.
- (iii) an expression that makes no sense mathematically.

(Write i, ii, or iii as an answer for each item.)

- (a) i / ii / iii $(A \cap C) \subset \overline{A} - C$
- (b) i / ii / iii $R \cap Q$
- (c) i / ii / iii $A + B - C$
- (d) i / ii / iii $(\exists x \in P)[x \in A \cup B]$
- (e) i / ii / iii $A \vee B \vee C$
- (f) i / ii / iii $B \Rightarrow (A - C)$
- (g) i / ii / iii $(A \subset B) \Rightarrow Q$
- (h) i / ii / iii $P \subseteq R \wedge Q$
- (i) i / ii / iii $P = Q \wedge R$
- (j) i / ii / iii $B \cap A - C$