# Math 300 – Homework 8

## Due Thursday 10/31 at the beginning of class

Total points: 141 (Writing portion: 45 pts (all the problems marked by $*$).)

## PART A

Problems from the textbook:

- Section 4.3

| problem | 1(a) | 2(a) | 4(a) | 5(a) |
|---|---|---|---|---|
| points | 6 | 6 | 6 | 6 |

- Section 5.2

| problem | 1(a) | 1(b) | 2 |
|---|---|---|---|
| points | 8 | 10 | 10 |

## PART B

1. Let $A = \{x, y, z, u, v\}$, $B = \{a, b, c, d\}$, and $C = \{5, 6, 7, 8, 9\}$.
   (a) [9 points] Write out three functions with domain $A$ and codomain $B$ making at least one of the functions have the property that its range coincides with its codomain (represent all functions by their graphs (see Definition 3 in notes) ).
   (b) [9 points] Write out two functions with domain $B$ and codomain $C$(represent all functions by their graphs (see Definition 3 in notes) ). Explain why you cannot define a function between these two sets for which the range equals its codomain.

2. Let $X = \{x \in \mathbb{R} | \ x \neq 7\}$ and $f : X \to \mathbb{R}$ be defined by $f(x) = \dfrac{2x + 5}{7 - x}$.
   (a) [5 points] Determine the range of $f$.
   (b) * [10 points] Prove that your answer for ran$f$ is correct.

3. * [10 points] Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 5x + 2019$. Prove that ran$f = \mathbb{R}$.

4. * [10 points] Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = -x^6$ and $S = \{y \in \mathbb{R} | \ y \leq 0\}$. Prove that ran$f = S$.

5. * [5 points] Let $f : [-1, \infty] \to \mathbb{R}$ be defined by $f(x) = \sqrt[10]{1 + x}$ and $S = [0, \infty)$. Prove that $S \subseteq$ ran$f$.

6. Express each of the following functions as a composition $f = g \circ h$. Be sure to give appropriate sets $A, B$, and $C$ such that $h : A \to B$ and $g : B \to C$. Note that neither $g$ nor $h$ should be an identity functions, but there may be many possible answers.
   (a) [7 points] $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sqrt[5]{e^{x^5} + 5}$
   (b) [7 points] $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \ln(x^4 + x^2 + 12)$
   (c) [7 points] $f : \mathbb{Z} \to \mathbb{R}$ defined by $f(x) = \cos(1 - \pi x)$

7. * [10 points] Let $f, g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = 2x^2 - 1$ and $g(x) = 3x + 5$. Determine $(g \circ f)(1)$ and $(f \circ g)(1)$.