

### 3. Linear Equations: Method of Integrating Factors (2.1)

1. A first order ODE is called **linear** if it is expressible in the form

$$y' + p(t)y = g(t) \tag{1}$$

where  $p(t)$  and  $g(t)$  are given functions.

2. If  $g(t) \equiv 0$  then

$$y' + p(t)y = 0 \tag{2}$$

is called a **homogeneous** linear ODE. Otherwise (1) is called a **non-homogeneous** linear ODE.

3. The equation (1) is separable if and only if there is a real constant  $\alpha$  such that  $g(t) = \alpha p(t)$ .  
(We already know how to solve it!)

4. The method to solve (1) for arbitrary  $p(t)$  and  $q(t)$  is called

*The Method of Integrating Factors*

### The Method of Integrating Factors

**Step 1** Put ODE in the form (1).

**Step 2** Find the integrating factor

$$\mu(t) = e^{\int p(t)dt}$$

Note: Any  $\mu$  will suffice here, thus take the constant of integration  $C = 0$ .

**Step 3** Multiply both sides of (1) by  $\mu$  and use the Product Rule for the left side to express the result as

$$(\mu(t)y(t))' = \mu g(x) \quad (3)$$

**Step 4** Integrate both sides of (3). Note: Be sure to include the constant of integration in this step!

**Step 5** Solve for the solution  $y(t)$ .

5. Consider

$$y' - 3xy = -xe^{x^2}.$$

(a) Find the general solution.

(b) Find the solution satisfying the initial condition  $y(0) = y_0$ .

(c) How do the solutions behave as  $x$  becomes large (i.e  $x \rightarrow +\infty$ )? Does that behavior depend on the choice of the initial value  $y_0$ ?