Math 220 – Homework 10

Due Tuesday 04/24 at the beginning of class

Total points: 169

PART A

Problems from the textbook:

- Section 5.5

problem	1	2	4*	5(b)	6(a)*	6(b)	10*
points	18	12	10	5	10	5	10

- Section 6.1

problem	1*	2*	3*
points	10	10	10

PART B

1. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=2020-4x$.
   (a) * [10 points] Compute $f([-4,4])$.*(Give a formal proof.)*
   (b) * [10 points] Compute $f^{-1}([-4,4])$.*(Give a formal proof.)*
2. Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x^2$.
   (a) * [10 points] Compute $f([-4,4])$.*(Give a formal proof.)*
   (b) * [8 points] Compute $f([-4,0])$.*(Give a formal proof.)*
3. [12 points] For each of the following functions write out $f(X)$ and $f^{-1}(W)$ for the given sets X and W, where $f:\mathbb{Z}\to\mathbb{Z}$.**(No proofs are necessary.)**
   (a)
   $$f(n)=\begin{cases} n+1 & \text{if } n\in\mathbb{E} \\ n & \text{if } n\in\mathbb{O}\end{cases}, \quad X=\{0,1,5,9\}, \quad W=\mathbb{O}.$$
   (b) $f(n)=n^2$, $X=\{-2,-1,0,1,2\}$, $W=\{2,7,11\}$
4. [3 points] Let S be a nonempty subset of $\mathbb{Z}^+$. Complete the following sentence:
   "An element a is not the smallest element of S if ..."
5. * [12 points] Prove the following so called Modified form of the Principle of Mathematical Induction:

   Let $P(n)$ be a statement about the integer n so that n is a free variable in $P(n)$. Suppose that there is an integer n_0 such that
   (a) The statement $P(n_0)$ is true.
   (b) For all positive integers k such that $k\geq n_0$, if $P(k)$ is true, then $P(k+1)$ is also true.
   Then $P(n)$ is true for every positive integer $n\geq n_0$.
6. [6 points] Restate the following so called Strong Principle of Mathematical Induction in set theory language.
   (`Hint: see the proof of the Theorem 1 in notes.`)

   Let $P(n)$ be a statement about the positive integer n so that n is a free variable in $P(n)$. Suppose the following:
   (a) The statement $P(1)$ is true.
   (b) For all positive integers k, if $P(i)$ is true for every positive integer $i\leq k$, then $P(k+1)$ is true.
   Then $P(n)$ is true for every positive integer n.
7. [8 points] Let $a,b,c\in\mathbb{Z}$. Determine the truth or falsehood of the following statements.
   (a) $\gcd(a,0)=a$.
   (b) Let a and b be not both zero. Then $\gcd(a,b)=\gcd(-a,b)$.
   (c) The Well Ordering Principle implies that the set $\mathbb{E}$ of even integers contains a least element.
   (d) $\gcd(a,b)=gcd(-a,|b|)$.