Math 220 – Homework 12

Due at the beginning of Final Exam.

1. Let $a = -255$ and $b = 143$
   (a) Use the Euclidean Algorithm to determine $\gcd(a, b)$.
   (b) Find integers x and y such that $ax + by = \gcd(a, b)$.
2. (a) Write the integer 42750 in a compact standard form.
   (b) Determine the following, representing your answer in the compact standard form:
   $$\gcd(2^{2018} \cdot 3^4 \cdot 55 \cdot 7^2, 6 \cdot 3^2 \cdot 77)$$
3. Prove that if p is a prime number greater than 3, then p is of the form $3k + 1$ or $3k + 2$.
4. Prove that if p is a prime number, then $\sqrt[n]{p}$ is irrational for every integer $n \geq 2$.
5. Prove or disprove that 3 is the only prime number of the form $n^2 - 1$.
6. Prove that if a is a positive integer of the form $3n + 2$, then at least one prime divisor of a is of the form $3n + 2$.
7. Let $a, b \in \mathbb{Z}$ with a and b not both zero. Prove that if $d = \gcd(a, b)$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.