Math 220/903&904-Homework 2

Due Wednesday 09/23 at the beginning of class

PART A

Problems from the textbook:

- Section 1.3 #1(b,c); 2(c); 4, 17
- Section 1.4 # 5, 6, 17[1], 20, 21

PART B

1. Given a quantified statement
$$\exists a \in \mathbb{Z}^+ \ni \forall b \in \mathbb{Z}^+, ab \in \mathbb{O}. \tag{1}$$
   (a) Express the given statement (1) in words.
   (b) Express the **negation** of the given statement (1) in symbols. **(Do NOT use the symbol $\notin$.)**
   (c) Express the **negation** of the given statement (1) in words.
2. Negate the following statements:
   (a) There is a cold medication that is safe and effective.
   (b) If x is a real positive number, then there is a real positive number ε such that $x < \varepsilon$ and $\frac{1}{\varepsilon} < x$.
3. Disprove the following statement: ``Let $n \in \mathbf{Z}$. If $n^2 + 3n$ is even, then n is odd.''
4. Consider the following statement:
   "If $\sqrt{3} < \sqrt{7}$, then $3 < 7$."
   Write in a useful form
   (a) the converse;
   (b) the contrapositive;
   (c) the converse of contrapositive;
   (d) the contrapositive of converse.
5. Prove the following statement:
   ``Let $n \in \mathbf{Z}$. Then n is odd if and only if $11n - 7$ is even.''
6. Prove the statement ``If n is an even integer, then $5n + 11$ is odd.'' by
   (a) a direct proof;
   (b) a proof by contrapositive;
   (c) a proof by contradiction.

[1]Hint: see Proposition 17 in the Lecture Notes(Chapter 1, part II)